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THE TERRITORIAL-PRODUCTION COMBINATION (COMPLEX) IN SOVIET ECONOMIC GEOGRAPHY†

by N. N. Kolosovskiy

1. INTRODUCTION

From the beginning of the great work of reconstruction, i.e., from the time of the first five-year plan, a new term, "the production complex" ("proizvodstvennyy kompleks"), reflecting a new phenomenon in our construction, has entered into Soviet economic and economic-geographic literature.

Still earlier, in the Gosplan (State Planning Commission) work on the economic regionalization of Soviet Russia, the notion of the region as a "production combine" ("proizvodstvennyy kombinat") had become current. The latter term, in the authors' intention, was to designate a profound production (technico-economic) combination of the various branches of the economy in the regions, primarily focussed around power. However, this term did not prove very satisfactory for designating territorial-production combinations arising in Soviet reality, and did not gain a foothold, partly, it appears, because of its insufficient flexibility in capturing the various shades of economic ties, which mean something quite different in their nature from a kombinat. But in a restricted sense the term "*kombinat*" has nevertheless become current, though not for every combination of lines of production.

It must be considered that these circumstances were the cause of the appearance of the term "production complex." It is still used to designate groupings of industries, both those linked by a common territory and those not thus connected. One speaks of "the complex of ferrous-metallurgy industries," regardless of the location of the individual branches of this complex; of "the Ural complex" of all the branches of industry in the vast territory of the Ural economic region; or of "the Novyy Tagil complex," or "the Lys'va-Chusovoye complex", located on more restricted territories.

From the very beginning the exact content of the term has not been established, and indeed it was hardly possible to do so because of the novelty of the planning and construction of mutually related groups of various industries, and because of the novelty of the economic and technical problems arising. Hence it is natural that various authors came to use the term to designate dif-

† The editors feel that this article, which first appeared in 1947 and was reprinted in *Principles of Economic Regionalization* (Osnovy Ekonomicheskogo Rayonirovaniya), (Moscow 1958), is significant both as an indication of the work of Russian geographers in the field of regional analysis and as an early approach to the concept of the industrial complex. The translation by Lawrence Ecker is reproduced with the permission of the American Geographical Society. It has been edited to conform with the space limitations of the Journal.

ferent concepts. By "territorial-production complexes" they came to understand, on the one hand, groups of enterprises organically related to one another, and, on the other, groupings that had almost no visible production ties with each other except a common territory, or the exploitation of the same natural conditions and common services, i.e., the possibilities, say of having a common supply of water or electricity or a common labor supply source. They came to speak of a "complex of branches of the natural economy" or a "complex of productive forces" alongside of a "complex (or integrated) use of raw materials and power," etc. Sometimes it is difficult to determine from the context what a given author means by such terminology. In a word, just as sometimes it is hard to distinguish the inscriptions and signs on a coin worn smooth by long circulation, even if they existed earlier, so too the terms "complex" and "integration" ("kompleksnost") do not now possess the proper degree of precision.

At the same time the knowledge and experience in socialist construction and the operation of enterprises have grown considerably, making it necessary to distinguish with sufficient precision all the diverse forms of mutual conditioning and co-subordination of ties and relationships for essentially different territorial-production groupings. From this naturally arises the need, especially in the scientific literature, for a more precise use of words.

2. THE COMBINATION OF PLANTS AND ENTERPRISES

In my article "*K voprosu ob ekonomicheskom rayonirovanii*"[1] an attempt was made to define the cognate concepts of the socialist kombinat and the complex. "By the production kombinat is meant the [profound] (the explanations in square brackets were added or expanded by the author for the latest edition) production combination or union of technological or energy processes in which two or more kinds of useful products (or results of labor) are obtained from one and the same substance. In comparison with specialized production processes, such a combination yields a definite economy of raw materials, energy, labor and transportation, and a reduction of production losses. The simplest examples of combination of production are the simultaneous production of steam and hot water for heating processes and of electricity in the thermo-electric centrals, and the "Orkla" method used to recover copper and elemental sulfur at the same time."

This definition, contains a number of inaccuracies, and it remains incomplete as it leaves out important questions of the combination of processes not only in one plant, but also in several plants and enterprises located in the immediate vicinity of one another in the same economic region. To develop the above, let us add that the chief feature of combination is the appearance of an additional product simultaneously in the material and in the value form.

The combination of specialized plants and enterprises, both those located very near to one another and those farther apart, is usually associated with the installation of intricate transportation networks for the transportation of raw materials and products, with the joint use of the so-called "wastes" of specialized enterprises, or with the joint use of sources of energy. Combination is an

[1] "Concerning the question of economic regionalization", ***Principles of Economic Regionalization***, N. N. Kolosovskiy, (Moscow 1958), pp. 60–86

important means of combating the dissipation of resources that takes place in the production processes even in the newest capitalist enterprises. In many cases we are for the time being unfortunately obliged to use these production processes in our Soviet plants as well, since the process of breaking out of the framework of capitalist production experience on a massive scale has only just begun. Here it should be noted that construction during the past three five-year plans has not been accompanied by a mass application of profound combination of plants, even those located on the same or adjacent grounds, unless these plants belonged to one and the same "official agency" (vedomstvo). What is more, plants counted as combined in the plans and projects have sometimes had to be "uncombined" in the process of construction.

Here there are two causes: one "in substance" and one "in form." "In substance"—let us note that in combining several different processes at separate plants it is necessary to reckon with both quantitative and spatial problems. If the distance of mutual transmission is small and the plants are small in the scale of their production, all the possibilities for combination are usually present and the undertaking becomes technically realizable and economically advantageous. But if we have to deal with an aggregate of plants of modern, very large scale, each of which occupies an area of 50—150 ha. or more (the "usual" size of the site of our metallurgic plant is 170 ha.), the distances between the plants and shops to be combined sometimes prove to be many kilometers, with the industrial areas of these "kombinats" extending for 10—15km. Meanwhile, the masses of wastes, by-products, steam, hot water, gases and other substances to be transported prove to be enormous. The organization of the transportation of these masses, sometimes passing or crisscrossing each other, proves to be a technically uncommonly complex task, sometimes doubtful as to its economic results, and sometimes downright unprofitable and not justified by the advantages of technological combination.

An effective means of facilitating the combination of plants is their production-flow (potochnyy) arrangement, such as was adopted, for example, in planning the coal-and-chemical kombinat at Cheremkhovo, according to the scheme: coal-concentrating station—electric station operating on concentration wastes—coking plant—organic synthesis from gases. This sort of arrangement in space, facilitating the successive transportation of the products and wastes of the various stages of production, can be recommended for a number of instances.

A second example of rational combination is the modern metallurgic plant, which consists essentially of several shops capable of existing separately, and actually so doing at several places: coal concentration—coking—electric central —furnace shop—open-hearth mill—rolling mill. The material and financial economy from combination has an energy "origin." It is determined by a unified energy economy with the use of all the thermal wastes (coke dust, fines, furnace and coke gases, etc.) and by the economy of heat (for heating metal) in various forms.

However, it is still necessary to supplement the definition of a production kombinat given earlier and include in it the case of the rational combination of several plants located near to one another, taking the above limitations into account. The category of spatial forms of combination further includes the various unions of regional power, joined with the respective networks: the union

of the electric economy on a regional scale, the union of the gas supply, etc., which are accompanied by fuel economies and by the raising of the total coefficient of efficiency of the system as against the isolated management of individual enterprises.

The Gosplan (State Planning Commission) term "regional combine" (*"rayonnyy kombinat"*) thus may have energy and transportation significance. With respect to the other elements of the production processes of the regions, it has a very restrictive interpretation for the present stage of development of the national economy. At the same time, for the broad perspectives of socialist construction, let us say for the stage of solution of the main economic task, the term may be retained in the broader sense, but again under certain technical and organizational conditions.

For a considerably broader circle of territorial groupings, the term "complex" and sometimes "territorial grouping", "node" (*uzel*), may be retained.

In the article quoted earlier we read the following definition of a complex:[2]

"The name of production complex is given to an economic (mutually conditioned) combination of enterprises at one industrial site or in a whole region, whereby a definite economic effect is achieved through a successful (planned) choice of enterprises in accord with the natural and economic conditions of the region, and with its transportation and economic-geographic situation."

This definition explains the term "complex" by the Russian term "mutually conditioned (co-subordinated) combination" ("vzaimoobuslovlennoye (sopodchinennoye) sochetaniye") in contradistinction to the simple co-existence of industries on a specific territory; the latter is better rendered by the word "grouping" ("gruppirovka"). (The term "complexness" or integration is best rendered in Russian by the words "mutual conditionedness', ("vzaimoobuslovemnost") or "mutual connectedness" ("vzaimosvyazannost").) It goes without saying that this implies a historical approach. Thus the territorial-production combination (complex) may in certain cases include both the economic formations of past historical epochs, especially those reconstructed by the Soviet power if they "combine" well with the economy of the region, and, new formations which, though they have not yet had time to become surrounded with all the "satellites", "nextdoor neighbors" or "stages" of the production processes, as well as with all the technico-economic ties with the existing economy of the given region, clearly have certain elements of the necessary ties.

In contrast to this, the spatial concentration of various industries which have either remained from the past or have arisen in the Soviet epoch and are without mutual ties or connections with the resources or economic-geographic situation of their region (even in the prospective future) cannot, from this point of view, be called a combination (complex). This is a mere territorial group of industries. Such cases of the appearance of "simple groupings" may occur also in a planned socialist economy; they arise because of natural or economic peculiarities of a given industrial location or because of conjunctural causes, or premeditatedly as the beginning of future industrial formations that are not yet completed.

The working out of the new economic regionalization of the USSR proposed by the government to the Academy of Sciences of the USSR has precisely the

[2] *Ibid.* p. 69

object of establishing the scientific grounds for a rational combination of the productive forces of the economic regions and the union republics in order to bring industry closer to the sources of raw materials and power and achieve the highest productivity of labor throughout the USSR as a whole, and also to achieve full economic equality of the nationalities as a basis for the development of the economy in the national republics. Thus, all the diverse forms of territorial-production ties must be involved.

The concept of the territorial-production combination, (proizvodstvenno-territorial'noye sochetaniye), provided it is subjected to a sufficiently profound study, will prove extraordinarily fruitful for economic-geographic investigation. It does not nor can it serve as a substitute for the doctrine of economic regions, nor does it "abolish" the regions themselves, but it does permit one to introduce organically into economic geography and regionalization the notions of productionally organized forces and forms of social labor, which act directly and consistently upon the process of the formation of regions and may modify both the internal structure of the region and its external boundaries and interregional relationships.

Whatever taxonomic units may be established by Soviet science for internal economic regionalization, it will be impossible to dispense with the notion of "cores", "foci" and "centers" forming the regions. It is impossible to reduce the process of regionalization to a mere fixing of the boundaries of the regions by formal methods or to a mere study of the geographic division of labor between the regions established almost arbitrarily (so-called "economicogeographic areas (arealy)", chosen for study under one circumstance or another).

The territorial-production combination, i.e., social human labor organized in definite techical forms, with its power and machine equipment, and applied to a definite combination of natural resources, is precisely the foundation of the geographic region-forming process, about which much has been said in the geographic science of our times.

The concept of territorial-production groupings and combinations (complexes) may be used in studying the process of formation of both of the major economic regions of Gosplan size and of the smaller taxonomic units of internal regionalization, all the way to an analysis of concrete groupings of production around one small urban settlement. The difference will consist solely in the degree of generalization of the production processes basic to the economy of a given combination (complex).

Generalization is admissible and justified when and if the study process can be generalized with its aid. It is also useful in cases where it is necessary to determine the processes not only qualitatively but also quantitatively, whereas the processes interesting us are too complicated for "direct" mathematization. These considerations hold in this case, since the ties between the combinations (complexes) are indeed extraordinarily diverse.

As may be seen from the foregoing, the kombinat may be regarded as a particular case of the complex, and the latter in turn as a particular case of the grouping. Production ties and general economic ties for regional complexes (combinations) may include the use of transportation, power, raw materials, semimanufactures, building materials, parts of machines and articles and labor force; and the use of common sources and installations of water supply, housing,

food and cultural and scientific forces and means; in a word, they are very multiform. A special place among them is occupied by "vertical" production ties, from raw materials up to the finished article: iron ore—pig iron—steel—rolled goods—castings—machinery—hardware; or coal—coking—coke-benzene chemistry—pigments—plastics, etc.

Ties also arise "horizontally", between the branches of the "vertical" series (e.g. coking—coking gas—open-hearth production; non-ferrous metallurgy—sulfur gases from roasting—sulfuric-acid chemistry—fertilizer chemistry, etc.). Farther down the line from the finished article (finished machine or complicated chemical product, etc.) there arise manufactories for making auxiliary parts, materials, chemicals, reagents, apparatus, instruments, and so forth, so-called "nextdoor neighbors" ("smezhniki") or "satellites" ("sputniki")—plants arising as "productional cooperatives", etc., owing to the necessity of having near at hand a whole "complex" of parts and apparatus for making this or that article of the "vertical" series.

3. CYCLES OF PRODUCTION

The multiformity of production ties of the regional economy explains why, in studying a concrete economy, an apparently irregular and externally extraordinary diversity of production combinations is obtained in each region taken separately. It is especially so when a comparison is made between regions, even those akin to one another in their type of economy. The geographer, accustomed mostly to grasping the characteristic peculiarities and individualities of regions, may at first find it difficult to discover in this diversity of regional production formations any foundation for their scientific classification and the solution of his typologic tasks. He must first of all do some analytical work, paying attention to the essence of the production processes themselves.

In all the multiformity of economic phenomena in the present-day industrial economy there are certain constantly recurring elements. This is the type of production process. Thus, for example, the production of ordinary pig iron in a modern plant is the same in the Donbass, the Urals, or the Kuzbass. This "type" process always requires coking coal and iron ore meeting definite specifications; and also reducing agents and ferro-alloys to obtain steel, refractory materials and fixing agents. Technologically, cement and glass plants are conveniently combined with this process, and with the coke economy it is convenient to combine cokebenzene chemistry, aniline-dye production, sometimes nitrogen and artificial liquid fuel production, sometimes soda production. In a certain part of metallurgical production, combination with machine manufacture is convenient, especially with heavy machine building and with metal-structure production.

The constantly recurring mass type of production process may be used as a basis in searching for the laws of region formation. A particular basic type of production process necessitates the selection of regions possessing the best combination of natural resources for it; coal, ores, fluxes, etc., for the integrated organization of the industrial economy.

Examining the whole "portfolio" of present-day manufactories, we can see that, beside the metallurgical cycle, there are also other "basic" processes for other kinds of mass raw materials and power. Analyzing these basic (leading)

mass production processes, we shall inevitably distinguish other extensive combinations of production processes which should cause the rise of other types of territorial-production combinations (complexes) in certain regions of our country.

Such is the path by which we approach the generalization of the concepts of production processes, which is necessary for an analysis of regional complexes and their parts. We need it for the consistent analysis of regional combinations (complexes) in economic regionalization. We shall, for the sake of brevity, give the conventional name of "energo-productional cycle" or simply "cycle" to the typical, constantly existing aggregate of production processes arising in a mutually conditioned manner around a basic processs, for a given kind of power or raw material. We note that a change of the basic process from one kind of power to another (for example, electric ferrous metallurgy instead of pyrometallurgy) effects a change in the composition of the manufactories of the cycle. Consequently, the cycle depends not only upon the basic raw materials, but also upon the kind of power used in the process and affecting the technology of the process.

Thus, by "energo-production cycle" is meant the whole aggregate of production processes successively evolving in an economic region on the basis of a combination of a given kind of power and raw materials, from the primary forms—extraction and refinement of the raw materials—to all kinds of finished products which can be produced on the spot, on the initial premise that the manufactory is brought near to the sources of raw materials and power and that a rational use is made of all the components of the raw-material and energy resources. In accord with the actual conditions of the region and the stages of development of the economy as a whole, this cycle may be full (completed) or not full (truncated) in individual regions. Each full cycle in the sense of the nomenclature of products may in turn yield either completed, finished articles only, or partly finished and partly uncompleted articles, parts of them or semi-products. The proportions between the one and the other are fixed in accordance with the economic assignments and the technological requirements in the line of planning. The cycle must be understood as a historical category, evolving in time.

To study the typology of regions and the system of regionalization, it is possible to establish the following generalized cycles:

1. A pyrometallurgic cycle of ferrous metals, including the mining of coking coal and iron ores, their concentration, the coking process, the blast-furnace process, the converting processes for the recovery of steel, the processes of rolling, steel and pig-iron casting, metal working, and machine building (primarily heavy, some medium). At the same time, it includes the use of coking products (coke-benzene chemistry, aniline chemistry, synthetic coal chemistry using gases and liquid coking distillates); plastics, motor fuel, nitrogen and hydrogen-containing compounds, solvents, alcohols, organic acids, rubber, high-octance additives to liquid fuel, etc. These often include inorganic chemistry, inorganic acids, alkalies, and fertilizers.

2. A pyrometallurgic cycle of non-ferrous metals, including the mining and concentrating of non-ferrous ores (copper, zinc, lead, gold, silver, and associated minerals), the extraction of coal or gases, the whole complex of pyrometallurgic processes of reduction of ores, the selection of metals and their refinement, the recovery of "rare" and "scattered" associates of non-ferrous metals, the recovery

of sulfur, sulfuric acid and chemical products on the basis of the use of sulfuric acid at its place of recovery; the working of non-ferrous metals and the recovery of alloys, the production of hardware and machine manufacture; cable production, involving a mass consumption of non-ferrous metals, the electric industry, and so forth.

3. A petroleum and energy-chemical cycle, including the extraction and refining of petroleum, gases, and salts. Here belong: the refining, cracking, and hydrogenation of petroleum and mazuth for the purpose of recovering motor fuel of different grades, lubricating oils, the organic synthesis of high-octane fuel and additives made from gases (natural and cracking gases), the organic synthesis of various kinds of rubber, alcohol, ether, organic acids, plastics, solvents, the recovery of hydrogen, nitrogen compounds, artificial fiber, the processing of various inorganic salts, the recovery of complex nitrogen-phosphorus and nitrogen-potassium fertilizers and alkalies, as well as other branches of the chemical industry developed on the basis of petroleum, gases, and salts.

4. The whole of the hydro-energy-industrial cycles on the basis of cheap mass electric energy with thorough introduction of electric power into both the main and auxiliary (heating) processes. This aggregate of cycles may include simultaneously or separately: electrometallurgy of ferrous, non-ferrous and light metals, electrochemistry (including hydro-electro-metallurgy), electrothermics (production of calcium carbide, electrophosphorus, cynanimides, alumino-cement, and so forth). The modern technique and practice of construction in regions with cheap mass hydro-power show that it is possible to construct great energy and production systems over extensive territories (five of the northwestern states of the USA, three provinces of Canada), with an extremely small participation of thermal (coal) energy by comparison with the usual norms, with most varied products, and with a sharp reduction in the expenditure of labor force as against the coal-and-metallurgy cycles. (These questions have been treated in more detail in the Angarstroy project). It should be noted that not every hydrostation is capable of causing changes of this sort in technology. Hydrostations with seasonal delivery of energy or playing a subsidiary role (in the amount of power output) in thermal electro-systems are unable to produce such changes in the scale and character of a whole economic region.

5. The whole of the cycles of the processing industry developing at a great distance from the sources of raw materials in regions with large populations and internal markets, and an advantageous economico-geographic and transportation situation, i.e., in regions of consumption. These cycles may develop partly on the basis of local kinds of raw materials and energy, but, for the most part, depend on fuel brought from elsewhere, transportable raw materials and semi-manufactures. This group embraces: the energo-mechanical, energo-chemical, and energo-thermal processing of mineral and organic raw materials and semi-manufactures, including machine building (medium, light, and precision); the ceramics industry; building materials production; the food industry; the textile industry and other branches of light industry with all the concomitant "nextdoor neighbors" and the waste-processing industry. So-called "secondary metallurgy" often arises here as such a "satellite". It is impossible to regard this cycle or aggregate of cycles as merely a part of the earlier named cycles, carried out into other regions, since by the very nature of its formation (convenience of

economic-geographic location) the regional cycle of the processing industry usually does not depend on any one raw-material region, but on several at once (for example, Moscow depends on the Urals, the Donbass, Middle Asia and so forth) and yet forms a usually distinct territorial "unity of multiformity", having as its basis highly skilled labor. In the "partial" forms, these cycles may also appear as concomitant ones in the regions where the basic cycles are developed (Donbass, Urals, and so forth).

6. The lumber-and-energy cycle has a substantial influence on the formation of the economy in a number of USSR regions. It includes forest exploitation, lumber processing (mechanical), the process of obtaining wood pulp, cellulose and plywood, the production of shingles, insulating slabs, pressed and glued lumber, the diverse chemical processing of wood, with the recovery of alcohol, ether, varnishes, organic acids, tanning agents, lubricating oils, rosin, turpentine and so forth. The cycle also embraces the production of the necessary thermal and electric energy from production wastes; charcoal metallurgy may be regarded as one of the branches of this cycle.

7. The whole of the industrial-agrarian cycles includes the various branches of socialist agriculture: field cultivation and animal husbandry, as well as the industry processing agricultural raw materials: the working of fiber (vegetable and animal), the leather industry, the processing of foodstuffs: flour and groats, wine, dairy products, sugar, delicatessen, tobacco, meat, fish; the soap industry; and so forth. The cycle also includes all rural industry operating on non-local and local raw materials and semimanufactures and utilizing the seasonal surpluses of agricultural labor. It likewise embraces the respective stationary and mobile energy bases.

8. The hydro-reclamational industrial-agrarian cycle is assigned to a separate group in view of its characteristic differences from the preceding ones. It includes hydro-reclamation works with the accompanying energy base, agriculture of the irrigation type with extensive distribution of industrial crops, and the processing industry.

All these cycles differ from one another in the characteristic relationships between expenditures of energy, raw materials, transportation operations, capital investment, and labor force for all the processes viewed together. Being dependent upon sufficiently stable technological foundations of the leading processes, the full cycles are sufficiently stable, i.e., they change relatively slowly in time owing to improvements in technology, and the balance relationships between the main production processes constituting the cycle quantitatively characterize their economic efficiency to a sufficient degree.

Thus, the economic regions can be characterized not only qualitatively, but also quantitatively by means of the cycle method (full cycles plus truncated cycles).

Since the eight energo-productional cycles, by their very design, exhaust all the varieties of processes, it is possible, with their aid, to characterize qualitatively and quantitatively any regional complex in the USSR, however multiform and peculiar it may be.[8]

[8] Theoretically, it is possible to foresee in the future the rise of a ninth cycle—atomic energy, obtained, as the literature shows, mainly in the forms of thermal and radiant energy, which can be converted into electricity.

However, here it is necessary to take into consideration a new circumstance. The fact is that in the mutual combination of manufactories of different cycles, new ties and new forms of mutual combinations, not previously considered, may arise and actually do arise between them.

Furthermore, every natural environment of a real complex, the composition of the raw materials, the peculiarities of the energy resources, the course and stage of the historic process of reconstruction of the region, the labor and ethnico-cultural peculiarities of the population—all this alters the type structure of the economy that can be logically obtained on the basis of the typical productional processes and combinations of them. Thus, something is created that is special, individual, and unduplicable at any other place, and that is typical for the genuinely live, ever developing economic region.

Hence, considerable differences between the "model" of the complex and the reality are not only possible, but are even bound to exist. Yet can it be said that the above intricate design and analysis of a stably existing complex, having the force of a law for the structure of the economy, have been made to no purpose? We have been interested in the typological problem of regional complexes (combinations) above all else, and less in an individual characterization of each complex taken separately. We have come quite close enough to this typological problem. It must still be taken into consideration that not all the lines of production of all the cycles are leading ones in the complex. Some of them are of local significance, others of nationwide significance. It is the latter that determine the "specialization" of the complex in the nationwide geographic division of labor. Moreover, it is positively necessary to take into account the stage of development of the complex, which affects the fullness of the cycles, and the rise of new cycles, which sometimes produce abrupt changeovers from one qualitative condition to a new qualitative condition.

4. TERRITORIAL-PRODUCTION COMPLEXES AND REGIONALIZATION

Since all the necessary preparatory analysis leading to the definition of the ties and laws governing them within the so-called cycles has already been given, it is possible to develop the notion of territorial-production complexes on a regional scale.

Already in the 1920's Soviet economic geography had abandoned the one-sided descriptive approach and the simple contemplation of such phenomena and facts as can be observed in the future or the present. Developing the historical scientific method in the geography of the socialist society, Soviet geographic science brought into the field of its attention the study of phenomena in the process of their development, including the future. Soviet geogrophy affirmed that the introduction into science of the evaluation of the prospects of development of regions is, in principle, wholly admissible without violating the objectivity of scientific research for the reason that under the Soviet system the plan of development of the national economy, built on a scientific foundation, supported by the masses and executed by a centralized economic apparatus organized into a governmental form, becomes an objective force. Hence, with a knowledge of the general laws of socialist development, the process of the formation of ecomic regions likewise becomes controllable. The study of these regions is the main task of the Soviet regional school of economic geography, and, as a science,

it has set itself the objective not only of perceiving reality in terms of space and region, but also of ascertaining the laws governing its planned transformation.

The teachings of the classicists of Marxism-Leninism regarding the basic laws of distribution of the productive forces under socialism have become a permanent part of the economic geography of socialism. The methods of economic regionalization for the immense territory of the USSR since the labors of the Gosplan, the M.I. Kalinin commission and the first and subsequent five-year plans have been worked out precisely on this basis. However, from the present-day point of view the degree to which the methods of regionalization in its concrete form have been developed leave something to be desired. The procedures which were suitable at the dawn of Soviet rule for a comparatively uncomplicated economic structure prove to be inadequate 25 years after the appearance of the project of economic regionalization of Soviet Russia (1922) in view of the enormous scale and multilateral development of industrialization, as well as in consideration of the prospects for a still broader growth of the economy in the stage of the solution of the main economic task. This can be seen merely from an analysis of the subject of territorial-production complexes.

Let us show the inadequacy of the methods now employed by means of examples and attempt to find ways of removing the defects. The investigation of the prospects for the development of the economy of any regional complex may be conducted either "from above"—from the general assignments of the union economy—or "from below"—from the possibilities of developing the productive forces of a given region, striving toward an optimum with respect to integration. The latter course in its extreme form (i.e., without the control assignments for the USSR) cannot lead to a positive result, if only for the reason that the number of variants differing in direction and scale can be very large; let us recall that the natural conditions and natural resources of themselves still do not decide anything; it is important to refract them through the prism of social relationships and requirements, and the latter are determined not only by internal needs but also by the relationships between the various parts of the state and its external political, economic, and military environment. Investigations "from above" are conducted by distributing the control assignment figures by separate branches of the economy and, in the subsequent phases—in the planning of plants and other enterprises—by the official planning and investigating organs, likewise "by branches".

The author performed the work originally by the first method ("from above"). In so doing, the levels of development of the USSR national economy were estimated by the method of per capita norms for 84 products (i.e. very broadly).

Then, the data on the natural resources according to official publications and the reports on the development of the economy of the individual regions were examined and data in research monographs on the major projects for the individual regions were evaluated insofar as possible. On this foundation, i.e. by no means mechanically or "statistically", the significance of the individual regional complexes in the USSR national economy was evaluated by leading branches of the national economy. In certain cases, the specialization and type of the complexes were determined in a fairly clear manner by this method, especially the complexes of heavy industry operating on coal energy and water

TABLE 1: Structure of the Economy of the Heavy-Industry Complexes Operating on Coal Energy and Hydro-Energy by Individual Branches of the Economy

Regional complexes	Coal mining and coking	Hydro-energy	Ferrous metal	Ferro-alloys	Copper	Zinc and lead	Light metals	Rare metals	Machine building	Basic chemistry	Petroleum extraction	Petroleum refining	Organic synthesis	Lumber industry (raw material)	Wheat	Meat	Textiles	Gold	Frequency of recurrence of types of production of different significance: +	0	−	Percentage of recurrence of + with respect to total number of types of production
Southern Ukrainian	+	+	+	+	−	0	+	0	+	+	−	0	0	−	+	0	−	−	8	5	5	44.5
Urals	0	0	+	+	+	+	+	+	+	+	+	+	0	+	0	0	0	+	12	6	−	66.6
Central Kazakhstan	+	−	+	0	+	+	0	+	0	0	−	0	0	−	0	+	−	+	7	7	4	38.8
Kuzunetsk-Yenisey[1]	+	+	+	+	0	0	+	+	+	+	−	0	+	+	+	0	+	+	13	4	1	72.3
Eastern Siberian[1]	+	+	+	+	0	+	+	+	+	+	0	0	+	+	0	+	0	+	13	5	−	72.3
Volga-Don[2]	+	+	+	0	−	−	0	−	+	+	−	+	+	−	+	+	−	−	9	2	7	50.0
Frequency of recurrence of an industry in structure of regions (+)	5	4	6	4	2	3	4	4	5	5	1	2	3	3	3	3	1	4	62	−	−	57.4
(0)	1	1	−	2	2	2	2	1	1	1	1	4	3	−	3	3	2	−	−	29	−	26.8
(−)	−	1	−	−	2	1	−	1	−	−	4	−	−	3	−	−	3	2	−	−	17	15.8
Percentage of recurrence of + with respect to total number	82.3	66.5	100.0	61.6	33.4	50.0	66.6	66.6	82.3	82.3	16.7	33.4	50.0	50.0	50.0	50.0	16.7	66.6	57.4	26.8	15.8	100.0

[1] The Kuznetsk-Yenisey (Middle Siberian) region possesses enormous natural resources of water power: the Ob'-Tom', the Upper Yenisey, the Lower Angara. In the 15-20-year perspective water power may play a most significant role in its energy balance, particularly the Yenisey-Angara station in the region of Yeniseysk. Consequently, for the broader prospect this is a region not only of the coal type, but also of the coal-and-water type. This applies to a still more marked degree to Eastern Siberia with its Angara River resources, which will be the chief basis of productional processes, thus influencing the type of the region.

[2] The Volga-Don region also has two "wellsprings" of energy: Eastern Donbass coal and natural gases.

(+) industries of unionwide or interregional significance;
(0) uncharacteristic branches and industries of regional significance;
(−) absence of the given branch of production.

power. But in the majority of cases the task proved so complicated that this method had to be abandoned after a number of attempts and a change made to that cycle method with preliminary grouping of production by interrelated "clusters" which was discussed earlier. As an example we give the tabular characterization of the complexes of heavy industry operating on coal energy and water power, as they were obtained from the first method. (see Table 1)

Thus, the prospective specialization and structure of the regional complexes according to the above-indicated method for the group of complexes of heavy industry present themselves in the following form:

1. *Southern Ukrainian Complex.* To the lines of production of interregional significance belong: coal mining, smelting of ferrous metals and ferro-alloys, machine building, basic chemical and cokebenzene industry, production of light metals. Among the lines of production of intra-regional significance are metallurgy of polymetallic ores, mining of rare metals, petroleum refining, and organic synthesis.

2. *Urals Complex.* Of interregional significance here are the smelting of ferrous metals (especially high-grade), ferro-alloys, copper, zinc, and light and rare metals; gold mining, machine building, petroleum extraction and refining, basic chemistry, lumber industry; among the industries of local significance for the Urals are coal mining and organic synthesis.

3. *Kuznetsk—Yenisey (Middle Siberian) Complex.* Among the industries of interregional significance are: coal mining, smelting of ferrous metals, ferro-alloys, light and rare metals; gold mining, basic chemistry, organic synthesis and lumber industry. Hydro-energy is of great significance. Uncharacteristic of the region are the recovery of copper, zinc, and lead and petroleum refining.

4. *Eastern Siberian Complex.* The industries of interregional significance are: coal mining, use of the hydro-energy of the Angara River, recovery of ferrous metals, zinc, lead, rare metals and gold; basic and organic chemical industry, and machine building; of especially great significance are the electricity-consuming lines: the smelting of light metals, electro-chemistry, electro-thermics. Copper mining and petroleum refining are not characteristic of the region.

5. *Central Kazakhstan Region.* To the industries of interregional significance belong: coal mining, smelting of copper, zinc, lead and rare metals. Uncharacteristic of the region are the recovery of light metals, machine building, basic chemistry and petroleum refining.

6. *Volga-Don Complex.* The industries of interregional significance are: ferrous metallurgy, coal mining, machine building, basic chemistry, petroleum refining, organic synthesis from petroleum and gases. Uncharacteristic are ferro-alloys and non-ferrous and rare metals.

Thus, typical features for all the complexes are coal mining on a mass scale for themselves and for shipment elsewhere (an exception is the Urals), the smelting of ferrous metals, heavy machine building (except for Kazakhstan), basic chemistry, non-ferrous metallurgy (in the Donbass its significance is more restricted), light metals (to a greater or lesser degree), organic synthesis—only for three complexes: the Kuznetsk-Yenisey, the Eastern Siberian, and the Volga-Don.

Least complicated is the structure of the Southern Ukrainian and Central Kazakhstan complexes. In contrast to this, the Urals, Kuznetsk-Yenisey, and

Eastern Siberian have an extremely complex outline.

These are the conclusions that can be obtained by the usual method of analysis: they might have satisfied us eight to ten years ago. But now they do not satisfy us, and here are the reasons why.

The mutually conditioned and interrelated nature of the industries listed in the table is not shown in any way. In the form in which given in the table, the complex appears as a simple "array" of industries arising or existing in a region more or less independently of one another. Still less visible are any laws governing a socialist economy or any distinctive features of planned regional complexes setting them off from the capitalist economy of analogous regions, for example the Ruhr or the American coal basins.

In the above formulation, only a few branches of the economy find a place, while numerous other branches of the economy developing in the region are, as it were, forgotten. Even if the author took them into account, they have disappeared entirely so far as the reader is concerned. An arbitrary element is thereby admitted into the typological characterization in the choice of index industries, and doubt is cast upon the scientific value of the method. If we recall that concern about the full-blooded integrated development of the economy of the regions rightly constitutes a special concern of the socialistic state, the importance of this fact will become understandable. At the same time it is quite evident that it is impossible to compile a table that would embrace 85–100 lines of production by name, still less all lines.

And, finally, it is quite impossible even roughly to characterize the economics of a complex by a well-defined system of quantitative indices under this system of study, without special and unwieldy investigations for each complex. All the more impossible is it to give a comparison between several variants of the development of one and the same complex, as may be required, for example, in an evaluation of given regional complexes in the system of economic regionlization of the Union as a whole.

5. APPLICATION OF THE CYCLE METHOD

The above defects, in the author's opinion can be, if not eliminated, at least greatly mitigated with the method of generalized energo-productional cycles. For in its very concept are laid the foundations for obligatory combination of interrelated industries and the preferability of arranging them in one complex (with specially motivated exceptions).

A characterization of the economy of the same complexes by the cycle method gives the following picture. (see Table 2)

The following conclusions with regard to the second method will become clear to all who attentively compare Tables 1 and 2:

1. The method is suitable only for studying the development of the economic regions of a socialist state; it is not suited to spontaneously developing capitalist regions.

2. The number of hypothetic assumptions in the evaluation of the tendencies and trends of development of complexes in the future is sharply reduced by the second procedure; consequently, the reliability of the predictions is increased. The typological features, and likewise the differences, stand out more vividly.

3. A general coverage of the structure of the economy is achieved instead

TABLE 2: Structure of the Economy of the Regional Heavy-Industry Complexes Operating on Coal Energy According to the Method of Energo-Productional Cycles

Regional Complexes	Ferrous pyrometallurgy	Non-ferrous pyrometallurgy	Petroleum-chemical	All hydroenergy-industrial cycles	All cycles of processing industry	Lumber-energy	All industrial-agrarian cycles	Hydro-reclamational-agrarian-industrial	Frequency of recurrence of different significance +	0	−	Percentage of recurrence of + with respect to total number of cycles
Southern Ukrainian	+	0	0	+	0	−	+	0	3	4	1	37.5
Urals	+	+	+	0	+	+	+	−	6	1	1	75.0
Central Kazakhstan	+	+	−	+	0	0	+	0	4	3	1	50.0
Kuznetsk-Yenisey	+	0	−	+	+	+	+	−	5	1	2	62.5
Eastern Siberian	+	+	0	+	+	+	0	−	5	2	1	62.5
Volga-Don	+	−	+	0	+	−	+	0	4	2	2	50.0
Frequency of recurrence of (+) cycles	6	3	2	4	4	3	5	−	27	−	−	56.2
(0)	−	2	2	2	2	1	1	3	−	13	−	27.1
(−)	−	1	2	−	−	2	−	3	−	−	8	16.7
Percentage of recurrence of + with respect to total number	100	50.0	30.3	66.7	66.7	50.0	83.4	0	56.2	27.1	16.7	100.0

Remarks regarding the Kuznetsk-Yenisey (Middle Siberian), Eastern Siberian and Volga-Don complexes are the same as those for Table 1.

of a selective one. Table 2 represents a kind of algebraic formula of the regions, embracing the laws governing each type of ties and the mutual co-subordination of the lines of production of the regions.

4. By using enlarged average indices for the cycles as a whole, it is possible to give approximate typological quantitative evaluations for the elements of a given type of national economy—the balances of requirements for raw materials, fuel, electric power, labor force, water, transportation services, as well as consideration for the demands for industrial land for plants, areas for forest exploitation, agriculture, etc. Furthermore, with a sufficient knowledge of the general geographic conditions of the region, it is possible to introduce the necessary (enlarged) corrections for local conditions and to obtain as a first approximation in case of need, a certain idea of the differences in the average regional social productivity of labor by comparison with the average conditions throughout the USSR, and to know in what sections of the national economy they may be expected. Here the balance estimates can be verified by the method of per capita norms.

5. Having this sort of characterization, one can, without doing violence to the logic of investigation, proceed to the next and more profound phase of typological study of the structure for the individual territorial components of the regional complex. One can distribute the general dimensions of production of each cycle separately over the territory of a given region in full accord with the geographic (natural and economic) conditions of that region without disturbing the integration and structure of the whole and consistently proceed to study more and more localized complexes (combinations), geographic groupings (nodes) and, in the course of investigation also kombinats of varying character. There is no doubt that in the second intra-regional phase of the investigation the degree of generalization should be lesser and that the distribution of production over the territory should pursue two objectives. First, we reduce, insofar as possible and advisable, the expenditures for the movement of raw materials, energy, finished products and other expenditures depending upon the geographic distribution of the economy on the territory of the regions. Second, is the task of reducing the total expenditures for the development of the territory, regional planning, and the construction of housing and enterprises. The choice of subregions and nodes should be made primarily in accordance with the sanitary, cultural, and esthetic requirements with relation to the location of the populated places.

In order to show the application of the cycle method to any type of regions, we choose the group of regional complexes developing on the basis of the use of petroleum, gases and hydro-energy. Whether by accident or not, these complexes turn out to be simultaneously hydro-reclamational, in which agriculture is being developed through irrigation of lands by means of larger or smaller public works, and these, in turn, are closely connected with hydro-energy. An exception here is the Western Ukraine. (see Table 3)

Comparing Table 3 with Table 2 as to frequency of the recurrence of the cycles, we see that the typical features stand out here no less vividly than in the preceding case. At the same time one is struck by the diversity of the economy of the complexes having a given trend and by their connection with agriculture.

Thus the prospective specialization and structure of the regional complexes

TABLE 3: Structure of the Economy of the Regional Complexes Developing on the Basis of the Use of Petroleum, Gas or Hydro-Energy According to the Cycle Method

Regional Complexes	Ferrous pyrometallurgy	Non-Ferrous pyrometallurgy	Petroleum-chemical	All hydroenergo-industrial cycles	All cycles of processing industry	Lumber-energy	All industrial agrarian cycles	Hydro-reclamational industrial-agrarian	Frequency of recurrence of cycles of different significance: +	0	−	Percentage of recurrence of + with respect to total number of cycles
Transcaucasian	0	+	+	+	0	0	+	+	5	3	−	83.4
Northern Caucasian	−	+	+	+	−	0	+	+	5	1	2	83.4
Central Asiatic	0	+	+	+	0	−	+	+	5	2	1	83.4
Middle Volga	−	−	+	+	0	0	+	+	4	2	2	66.7
Western Kazakhstan	−	−	+	−	−	−	+	+	3	−	5	50.0
Western Ukrainian	−	−	+	0	0	+	+	−	3	2	3	50.0
(+)	−	3	6	4	−	1	6	5	25	−	−	52.1
Frequency of recurrence of cycles (0)	2	−	−	1	4	3	−	−	−	10	−	20.8
(−)	4	3	−	1	2	2	−	1	−	−	13	27.1
Percentage of recurrence of + with respect to total number	0	50	100	66.7	0	16.6	100	83.4	52.1	20.8	27.1	100.0

of the petroleum-and-hydroenergy group is determined as follows by the cycle method:

1. *Transcausian.* To the cycles of interregional significance belong: the non-ferrous metallurgy cycle—copper, rare metals; the petroleum-chemical cycle—extraction and refining of petroleum, petroleum chemistry including organic synthesis; among the hydro-energy cycles—ferro-alloys, aluminum, carbide production and its derivatives, particularly rubber; the industrial-agrarian cycle, represented by the production and processing of subtropical crops, tea, citrus fruits and so forth; the reclamational cycle—plantings of rice, cotton and other crops on irrigated lands. Of intra-regional significance are the cycles of the processing industry, for example textiles, ceramics, machine building; and the lumber-energy cycle—mountain forests.

2. *Northern Caucasian.* This complex stands out vividly as very much akin in structure to the Transcaucasian regional complex, with the difference that zinc and lead appear in place of copper, and northern agricultural crops in place of subtropical ones.

3. *Central Asiatic.* This stands out no less vividly as an analogue to Transcaucasia, but with reclamation having much greater importance and petroleum extraction less pronounced significance (one must suppose, prior to the proper prospecting).

4. *Middle Volga.* Of great importance for this region is the petroleum-chemical cycle and gas, and the extensive development of chemicals from petroleum derivatives and salts (also from potassium salts), including organic synthesis. Furthermore, hydro-reclamation and hydro-energy are important in the Transvolga.

This complex may quite rightly be assigned to the kindred type of regions. Without doubt, the existing differences from the other more southerly regions are to be explained by two causes: the different geographic location in more northerly latitudes and zones, and different geologic origin: the Volgaland is a part of the Russian Platform, while the earlier mentioned regions lie in the folded zones of the continent. These circumstances affect the individual face, but not the type.

5. *Western Kazakhstan.* From the production viewpoint, this is the extreme expression, the accentuation of the qualities peculiar to the Middle Volgaland; petroleum and salts determine the great development of the petroleum-chemical cycle and the processing of salts, including potassium salts; irrigation of the land is here the only means of cultivation (see the projects for irrigation with water from the Ural River). The agricultural cycle is for the time being represented by migratory forms of cattle raising.

6. *Western Ukrainian.* Geographically this would seem to be the exact opposite of the Western Kazakhstan Complex. However, in this representative of the group there are important forms of kinship with it in the field of industry (petroleum extraction, gas extraction, refining of petroleum and gases, the water resources of the mountain rivers, ore resources, exploitation of salts, with the development of the respective cycles on these bases). There are features of similarity to the Caucasian regions, explainable to a certain extent by the similarity of the geologic basement (young fold zone). The economic-geographic differences from Western Kazakhstan are rooted in the conditions

for agricultural production, which is represented in diametrically opposite forms (intensive farming and migratory cattle raising), affecting the density of settlement and the industrial "summit" of agriculture—the food and light-manufacturing industries of the complexes, which are, naturally, also very different.

From an examination of this example of characterizations of the structure of the regional complexes of the petroleum-hydroenergy group, as well as on the basis of the earlier example for the complexes of heavy industry, there follows much that is important from the theoretical viewpoint about the nature of such complicated formations as are regional complexes. They appear before our eyes not as the mechanical sum of the productional cycles or processes (let us remember that the concept of cycle has been introduced by us as an auxiliary one); the regional complexes should be examined and studied as a unit of multiformity, as phenomena developing according to all the socio-historical laws.

Thus far in our constructions we have, as it were, ignored the natural geographic environment. It has entered into our arguments in a hidden form: as the resource of industrial raw material obtained at a given place; as the energy of a production process from a natural source located nearby; as a resource for industrial-agrarian processes on the basis of agricultural raw material obtained in a given region with all its peculiarities of climate, soils, and vegetation. But the natural environment has thus far not been specifically considered. When we spoke of the heavy-industry complexes, this defect was not so striking, but when we passed on to the complexes of the second group, we needed to explain a number of the circumstances that prompted us to regard as legitimate the assignment of all six regions to the given group. The forces of nature, of the geographic environment, as such, broke forcibly into our theoretical construction and compelled us to speak about them.

Earlier, we properly assigned first place in our construction to questions of production, (i.e., economic and technical questions affecting the structure of the regional complexes), but we forgot such forces of nature as solar energy, wind, soil and so forth, which, rendering gratuitous service to man exert a rather strong influence upon the structure of production of regional complexes. Their organic inclusion in the typologic scheme along with the forces of technological processes (electricity, heat, steam, chemical energy and so forth), is obviously quite indispensable for the proper completeness and correctness of economico-geographic conclusions.

Further, it must be taken into account that the number of different combinations from our eight to ten conventional generalized cycles may be very large. The quantitative relationships between the industries of the different cycles may be almost infinitely varied. Hence, within one and the same grouping of regions (heavy industry, petroleum-hydroenergy, etc.) there may be a fairly broad range of quantitative-qualitative deviations in one direction or another. There may (and do) appear intermediate forms, "hybrids", so to speak, which it will be difficult to assign to the one or the other type.

Thus, for example, in our first grouping of the "heavy-industry" complex the Donets (Southern Ukrainian) complex, based on coal energy, stands out quite distinctly as the "classical" complex. Differing little from it with respect to energy is the Central Kazakhstan Complex (deviation toward non-ferrous metallurgy); the Kuznetsk-Yenisey Complex just now appears as a "sort of

Donbass in Siberia", but in the prospective future, for example when the hydrostation on the Yenisey River, capable of furnishing the equivalent of the energy now furnished by the coal mined in the Kuzbass, is put into operation, the circumstances must change sharply; new processes will be created, not on the basis of the thermal, but the electric form of energy, i.e., new cycles will appear with equal rights; the complex will appear before us as a coal-hydroelectric one.

The Eastern Siberian Complex since the first steps of its development has appeared as a principally hydro-energy one.

The Volga-Don heavy-industry complex inclines in another direction from the Donets coal "standard." This is also a transitional type to the complexes of the petroleum-gas-hydro-energy series. On the one hand, it is the same Donbass in its eastern part, but, on the other hand, it is the salt, gas, and petroleum bearing Volga in its new, present-day appearance. Thus, there are to be several transitional types within the "family" (group of types) of the heavy-industry complexes and of course, as we have seen earlier, also within the "petroleum-hydroenergy 'family'".

It is possible, with an attentive study of the structures, to arrange the complexes of the two "families" in a successively changing series forming, in the transition from one regional complex to another, certain "jumps" from one measure of things (quantity—quality) to another measure of things, depending upon the relationships between the dominant[4] forms of energy (and raw material) and upon the related forms of productional cycles having different compositions. The difference in the forms of natural (elemental) energy affecting the production processes (for example, in agriculture, lumbering and hydro-energy) must also be taken into account here.

It turns out that the size of the "jumps" in the transition from one region to another within one "family" generally is not greater than in the transition from one extreme member to another extreme member of the two "families." Calling, for brevity's sake, the dominant types of energy: h hydro-energy, c coal energy, p petroleum and gas energy, and arranging the regions in accordance with the transitions from one dominant energy to another, we obtain the following series:

1) Eastern Siberian; h, transitional to the Far Eastern and Northern Yenisey complexes;
2) Kuznetsk-Yenisey; $c + h$;
3) Central Kazakhstan with the Altay; $c + h$; without the Altay; c;
4) Southern Ukrainian; c;
5) Volga-Don; $c + p$, transitional to the Middle Volga and Western Kazakhstan complexes;
6) Urals; $c + p$, transitional to the Middle Volga and Western Kazakhstan complexes;
7) Western Kazakhstan; p;
8) Middle Volga; $p + h$;
9) Western Ukrainian; $p + h$;
10) Middle Caucasian; $p + h$;
11) Transcaucasian; $p + h$;
12) Central Asiatic; $p + h$, transitional to the Southern Kazakhstan complex.

[4] Translator's note: *dirizhiruyushchyy* "conducting (e.g. an orchestra)" here apparently means "dominant".

Analogously to the procedure employed in distinguishing the two "families" of complexes, heavy-industry and petroleum-hydroelectric, all the other regional complexes can be grouped in three new "families" marked by the same characteristics of mutual successive transitions of structure and cycle composition. Let us enumerate all the "families" in full:

1) heavy industry,
2) petroleum—hydro-energy;
3) industry—farming;
4) processing industry;
5) northern industry.

Let us characterize all these "families" briefly:

1. *Heavy industry.* The complexes specialize in the mining of coal and the smelting of ferrous and non-ferrous metals by pyrometallurgic methods. In the transitional forms of more complex composition, the regions include mass metallurgy operating on hydro-energy by electrometallurgic methods. The complexes possess a ramified system of machinebuilding and chemical industries, which arise in them on the basis of the principle of orientation to the bases of raw materials and energy and the reduction of expenditures for transportation and energy. The complexes have great individual differences depending upon the quality and quantity of the basic raw material being processed.

Within the group are distinguished principally coal-and-metallurgy complexes, complexes with principal development of non-ferrous metallurgy, complexes in which ferrous and non-ferrous metallurgy have approximately equal significance, complexes with principal development of mass water power (with insured regulation) and electro-processes. The chemical industry of the complexes varies according to the presence or absence in their territories of petroleum, natural gases and lumber, as well as according to the degree of diversity of the mineral chemical raw materials.

2. *Petroleum-hydroenergy.* These complexes specialize in the extraction and refining of petroleum and gases and the production of water power. In the present-day industrial economy petroleum and natural gases occupy one of the leading places as sources of energy and as chemical raw material. All agriculture, motor and air transportation now have petroleum as a source of motive power. Electric energy obtained from gas occupies in value a middle place between coal and water power. Gas is transmitted cheaply for long distances through pipelines. Petroleum and gases yield an immense quantity of organic synthetic products. Hydro-energy creates an exceptionally favorable combination with petroleum resources, widening the the range of electricity-consuming processes both into the field of organic synthesis and into that of the adjacent electrochemical, electrothermal, and electrometallurgic industries. The ramifications of production are very great, and are still growing.

Under USSR conditions the respective complexes of natural resources (petroleum—hydro-energy) coincide, as a rule, with localities rich in agricultural reserves of unionwide significance requiring hydro-reclamation. Hydro-reclamation and hydro-energy require unity of the water economy as a whole and consequently are closely connected with one another with respect to production, both in the creation of hydro-works and in their operation. The complexes of this group also differ greatly from one another depending upon the character

of agricultural production, the relationships of the cycles, upon the type and intensity of the physical-geographic processes.

The extreme members of this "family" come close in type, on the one hand, to the heavy-industry group and, on the other, to the industrial-agricultural group.

3. *Industry—Farming.* A characteristic feature of the complexes is the presence of a big industry processing the foodstuff and technical raw materials produced by agriculture (grain, root crops, fruits, fiber, meat, leather, wool, and so forth). In the USA the food industry occupies first place in value. Hence the importance of the branches of this industry in the national economy of industrial-agrarian countries. However, the industry of these complexes is not confined to these forms. In a number of complexes located in the forest-steppe belt there appear lumber cycles of local significance; in the maritime regions, i.e., fishing regions, fish processing appears; in the complexes advantageously located with respect to the heavy-industry complexes—light and medium machine building; in the maritime—shipbuilding and so forth.

Thus, being close to the petroleum—hydroenergy complexes (through agricultural production, hydroelectric cycles, reclamation), the given complexes are transitional (through machine building) to the following group—the processing industry, which also consumes a considerable amount of raw material of agricultural origin.

4. *Processing Industry.* The group of complexes of the processing industry, which, as a rule, are located in the more northerly latitudes with a climate having a less intensive energy, is characterized by the high significance of the processing branches of industry, including those producing the means of production from the semimanufactures of the heavy-industry regions. Here may be represented the most varied "upper stages" of the metallurgy cycles, the cycles of the mechanical, thermal, and chemical working of articles, as well as the textile, lumber, and foodstuff industries.

Processing industry develops with special success in localities with a dense population and large local consumption, as well as in localities marked by advantages of economico-geographic position, which permit economies in the transportation of the finished products. The modern technology of refining (enriching) certain kinds of raw materials reduces the "inert material" in the composition of the raw materials, furnishing transportable "concentrates." The same object is attained by combining production processes with the recovery of useful products from yesterday's "inertial material", which also enlarges the sphere of supply. These circumstances determine the advisability and advantageousness of the development of processing cycles in the central regions of the country with their more favorable economico-geographic location, and likewise the advantageousness of developing certain "basic" industries affording maneuverability (ferrous metallurgy near Moscow and Leningrad, and so forth).

It is clear that any change in the economico-geographic position must affect the intensity of the development of these regional complexes. For example, the growth of production in the Urals and Siberia assumes important stimulating significance for the Volga—Kama regions, improving their supply of eastern semimanufactures. The establishment of economic ties with the countries of the new democracy in the West improves the economico-geographic position of

TABLE 4: Table of Regional Territorial-production Combinations

Forces of productive processes						Forces of natural processes		
Coal production (% of unionwide production)	Consumption of electric power (% of unionwide consumption)	Per capita norms of electric-armament, with 100 taken as unionwide average	Directing types of energy	Groups of complexes ("families")	Name of regional combination (complex)	Type of air masses	Zonality	Tectonics
–	1.5	250	h	Northern industry	Karelian-Murmansk (transitional)	ar+at	t+f	pl
2.0	0.5	400	h+c		North-Yenisey	ar	t+f+vz	pl
0.5	0.8	200	h+c+p		Yakut	ar	t+f+vz	pl+fz
4.9	1.2	150	c+p		Dvina-Pechora (transitional)	ar+at	t+f	pl+fz
–	0.3	100	c		North-Ob'	ar+at	t+f	fz
0.5	0.3	100	c+p		Chukot-Kamchatkan	ar−p	t+vz	fz
4.2	4.0	200	c+p+h	Heavy industry	Far Eastern (transitional)	ar−p	f+vz	fz
6.3	7.3	300	h+c		Eastern Siberian	ar	f−vz	pl+fz
16.6	10.3	250	c+h		Kuznetsk-Yenisey	at−ar	f+fs+vz	fz+pl
6.7	2.5	250	c+h		Central Kazakhstan with Altay	at	s+d+vz	fz
29.2	8.1	150	c		Southern Ukrainian	at	s	pl
6.7	3.9	150	c+p		Volga-Don (transitional)	at	s	pl
6.7	8.7	150	c+p		Urals (transitional)	at	s+fs+f+vz	fz
0.5	0.6	100	p	Petroleum-Hydro-energy	Western Kazakhstan	at+tr	s−d	fz
–	3.7	75	p+h		Middle Volga	at	s+fs	pl
1.3	1.9	60	p+h		Western Ukrainian	at	f+fs+vz	pl+fz
1.7	3.5	110	p+h		Northern Caucasian	at	s−vz	fz
1.7	4.4	150	p+h		Transcaucasian	at+tr	vz	fz
2.5	4.8	90	h+p		Central Asiatic	tr	d+vz	fz
0.5	0.8	50	h		Southern Kazakhstan (transitional)	tr+at	d+vz	fz
–	1.2	40	h	Industry-Farming	Black Sea with Moldavia	at	s+vz	fz+pl
1.7	2.8	50	h+c		Central Ukrainian	at	s+fs+f	pl
–	0.4	60	c		Northern Kazakhstan	at	d+s+fs	fz
–	1.9	70	c		Western Siberian	at	s+fs	fz
–	3.1	70	c		Central Chernozem (transitional)	at	s+fs	pl
–	5.8	100	c	Processing industry	Volga-Kama	at	fs+f	pl
5.8	8.5	100	c		Moscow	at	fs−f	pl
–	1.6	60	c		Belorussian	at	f	pl
–	1.6	60	c		Baltic	at	f	pl
–	4.0	100	c+h		Leningrad (transitional)	at	f	pl
100%	100%	100	c+h+p					

SYMBOLS

I. Dominant types of energy:
- c coal
- h hydro-energy
- p petroleum and gas

II. Types of air masses:
- ar arctic
- at atlantic
- p pacific
- tr tropical

III. Zonality:
- t tundra
- f forest
- fs forest-steppe
- s steppe
- d deserts
- vz vertical zonality

IV. Tectonics:
- pl platform
- fz fold zone

Belorussia and the Baltic lands. We note that the good moisture supply and dense hydrographic network for the supply of water to industry and the large cities is of importance for the successful development of all the regions of this group.

5. *Northern Industry.* The complexes of this type are located in the forest and even tundra zones of the northern regions. The population here, as a rule, is sparse owing to the difficult climatic conditions. Under the conditions governing the conquest of territory, only the selective method of developing lands by separate oases and areas is possible, as a rule. It is often necessary to rely not only on permanent cadres of the local population, but also on the periodic importation (for definite terms) of labor force from other, more southerly regions of the country. The principal aim of creating a complex is the working of especially valuable mineral resources, forests, and the wealth of fish and other sea animals. Modern technology and economics permit the development of this northern wealth through especially high mechanization of the processes and the shifting of the processes to such forms of energy as water and oil power, which make it possible to reduce sharply the expenditure of live labor. Within the group there are considerable individual deviations of complexes, so that it is possible to establish transitional forms to the regions of the processing industry, on the one hand, and to the heavy-industry regions, on the other.

We can now compile a synoptic typological table (see Table 4) of regional production combinations (complexes) for the stage of the solution of the main economic task of the USSR. This table embodies as a classificatory feature the above grouping of complexes into five "families". In addition, the *variability of the complexes* of one and the same "family," brings different "families" together so as to produce a sort of continuous series of transitions, including 30 regional complexes of the USSR.

To the left and right of the name of each complex in Table 4 are placed several characterizations of the forces of the production processes and the forces of the natural processes operating in the complexes, viewed from the angle of their significance for the economy of the complexes. Taken as the indices of the forces of the productional processes are: coal mining (in percentages of unionwide production), electro-energy consumption (in percentages of unionwide consumption), norms of per capita electric equipment of labor (with 100 as the average for the USSR) and the dominant types of energy (coal, petroleum, water power). Taken as indices of the forces of the natural processes are: the type of air masses and zonality—for the solar forces; the main forms of tectonic types (platforms, shields, fold zones)—for the plutonic forces affecting the formation of relief, stratigraphy, the creation of magmatic "foci", the geochemical processes, the laws governing the concentration and dispersion of the elements in the earth's crust, and so forth. The subjection of these indices to analysis is grounded on the following assumption. If the arrangement of our complexes in a successively changing series is founded only on a logical basis, i.e., is an abstraction having no foundation in reality or in the laws governing the regional structure of a socialist economy for such a vast and diversified country as the USSR, then our energy indices for the natural and technological forces should reveal this in the form of complete casualness and inconsistency in the series of these indices. But if we discover a law-governed variation of these auxiliary

series in any form, we shall obtain a new and important proof that the structure of the regional complexes is subject to some law of development.

As we see from Table 4, there are a number of grounds for believing that such a law does exist, and that it leads to the following conclusions:

1. The highest electric armament of labor (intensity of productional processes) is obtained for the complexes of northern industry (North Ob'; Chukot—not an exception, but only a later phase of inclusion in the process of industrialization, which has not yet had time to evolve to the estimate stage).

2. The following places, as was to expected, are occupied by the heavy-industry regions, with the new, thinly settled Siberian regions holding the highest place with respect to electric armament.

3. Then come the petroleum-hydroenergy complexes—new kinds of heavy and medium industry, combined with agricultural irrigation installations.

4. The fourth and fifth places are occupied by the complexes of industrialized farming and the processing industry, requiring by their nature a considerable expenditure of live labor.

5. The types of air masses change too consistently not to see in this series a confirmation of the correctness of the assumed sequence of complexes. Nor are any substantial contradictions encountered in the "zonality" column.

6. The "tectonics" column affords no materials for judgment without a parallel examination of the geochemical laws, for which it is as yet difficult to give generalizing index. However, in a number of cases, it helps to verify the location of the complexes within a "family" (it is preferable to place the Eastern Siberian complex along with the Kuznetsk-Yenisey, not otherwise; the Volga-Don complex with the southern Ukrainian; the Middle Volga complex along with the Western Ukrainian; the Black Sea complex with the Central Ukrainian, and so forth).

7. It is to be expected that further study of the question from the viewpoint of physical-geographic, geologic and geochemical regionalization, continuing the scientific traditions of V.I. Vernadskiy and A.E. Fersman, will help to clarify the many problems which remain.

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MARKOV CHAINS, EXCHANGE MATRICES, AND REGIONAL DEVELOPMENT

by Paul E. Smith*

In light of the recent attention paid to the appearance and discovery of so-called depressed areas in the United States, and to cold war developments in Latin America and Southeast Asia, it has become understandably fashionable among economists and political decision makers to be concerned with regional economic development. Unfortunately, it is not always clear just what is meant by either "depressed" or "region," although the former term is usually taken to imply that the area in question has a lower per-capita income than other similarly defined areas. We shall adopt this definition for the lack of a better one and avoid the political, geographical, and economic pitfalls of the second problem by merely defining a region as a bordered topological space. Hence, we specify that the inhabitants of some regions enjoy a higher level of living than the residents of others and that it is deemed socially desirable to narrow the income gaps between regions.

Two broad types of solutions, both entailing the geographic reallocation of resources, are often proposed. The first or market solution is in the classical tradition and suggests that, if impediments to labor mobility are absent, human resources will move from the low-income regions to the high-income regions until the income differentials within competing groups are wiped out. In the case of noncompeting groups, public subsidized training programs are sometimes advocated, e.g., West Virginia coal miners are converted into California real estate brokers at government expense. However, recent legislation, possibly due in part to lobbying activities by "California real estate brokers", has been aimed in the direction of subsidies to the inhabitants of depressed regions and the raising of the income base in those areas via capital investment.

The purpose of this paper is to explore the possibility of increasing the incomes of some regions relative to those of others. We shall not be concerned so much with the techniques of growth *per se*, but rather with whether such development can be attained within the framework of a model of interregional trade. For this purpose we will find it convenient to employ stochastic or Markov matrices as a useful mathematical tool. The problem will be briefly considered first from the standpoint of comparative statics and then within the context of a dynamic model. Numerical examples will be used extensively.

1. COMPARATIVE STATICS

Consider n geographic regions $R_1, \cdots, R_n$ which may or may not trade with each other. Each region R_i has income Y_i which it spends either on its own

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output or the output of other regions, with domestic consumption and imports having linear and homogeneous relationships with its own income.[1] Hence a_{ij} may be said to be the marginal and average propensity of R_i to spend in R_j. Therefore, we can write the exchange matrix

$$A = \begin{pmatrix} a_{11}a_{12} \cdots a_{1n} \\ a_{21}a_{22} \cdots a_{2n} \\ \cdots\cdots\cdots\cdots \\ a_{n1}a_{n2} \cdots a_{nn} \end{pmatrix}$$

Clearly A is a square matrix with nonnegative elements and with row sums equal to unity, i.e., $a_{ij} \geqq 0$ and $\sum_{j=1}^{n} a_{ij} = 1$. This is because no region can make negative expenditures in any region and in equilibrium each region spends all of its income. Therefore A may be called a stochastic or Markov matrix of transition probabilities.

We shall call the subset of regions $R' \subset R$ closed if no region in R imports from any region in $(R - R')$ so that $a_{ij} = 0$ for $i \in R'$ and $j \in (R - R')$. If $a_{ii} = 1$, then R_i spends all of its income on its own output and is said to be an absorbing state or region. Moreover if R contains more than one closed set, the rows and columns of A can be rearranged into A^*. A^* is partitioned with closed sets of coefficients along the main diagonal, i.e.,

$$A^* = \begin{pmatrix} A_1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & A_2 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & A_3 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & A_k \end{pmatrix}$$

where the $A_i (i = 1, \cdots, k)$ are indecomposable or irreducible subsets of coefficients of the decomposable or reducible parent set A.

The total income of each region R_i is given by summing its sales to itself and all other regions, i.e.,

$$y_i = \sum_{j=1}^{n} a_{ji} y_j \tag{1}$$

Hence an equilibrium income vector is a vector $y = (y_1, y_2 \cdots, y_n)$ which satisfies the equation

$$A'y = y$$

or

$$(I - A')y = 0\,, \tag{2}$$

where I is the identity matrix of order n. Equation 2 is seen to be equivalent to the set of n homogeneous linear equations

$$(1 - a_{11})y_1 - a_{21}y_2 - \cdots - a_{n1}y_n = 0$$

[1] We assume that prices are held constant so that all variables are expressed in real terms.

$$
\begin{aligned}
&-a_{12}y_1 + (1-a_{22})y_2 - \cdots - a_{n2}y_n = 0 \qquad (3)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&-a_{1n}y_1 - a_{2n}y_2 - \cdots + (1-a_{nn})y_n = 0\,,
\end{aligned}
$$

whose matrix is $(I-A)$. A basic theorem of linear algebra asserts that a nonzero solution vector y of these equations exists if and only if $(I-A)$ is a singular matrix. We assume this to be the case and concern ourselves only with whether y is non-negative as well. That such an equilibrium vector exists and that all of its elements are positive has already been proven by Gale [3]. For a more general discussion of existence and uniqueness theorems, see Baumol [1].

If A is indecomposable, the equilibrium income vector y is trivially nonunique in that any scalar multiple of a solution vector is also a solution vector. If A is decomposable with indecomposable subsets $A_1, A_2, \cdots, A_k$ and if y is an equilibrium solution vector for A, then $y = (y_1^*, y_2^*, \cdots, y_m^*, \cdots, y_k^*)$ where y_m^* is an equilibrium solution vector for the set of regions with the non-decomposable submatrix A_m. That is, a solution vector can be found for each submatrix, and these can be suitably ordered so that they are also solution for A. Furthermore, a vector, say cy^*, found by multiplying any solution subvector by a positive scalar c is also an equilibrium solution vector. Hence for decomposable matrices the general solution vector is nonunique from the standpoint of relative incomes since any non-negative linear combination of the subsolutions is also a solution.

A simple numerical example may help to illustrate the problem. Consider the three regions represented by the indecomposable matrix

$$
A = \begin{pmatrix} 0.5 & 0.2 & 0.3 \\ 0 & 0.6 & 0.4 \\ 0.2 & 0 & 0.8 \end{pmatrix}
$$

Inasmuch as each region receives its income from sales either to itself or to other regions, the equilibrium income vector y can be found by simultaneously solving the system of homoneous linear equations

$$
\begin{aligned}
0.5\,y_1 \qquad\quad &+ 0.2\,y_3 = y_1\\
0.2\,y_1 + 0.6\,y_2 \qquad &= y_2\\
0.3\,y_1 + 0.4\,y_2 + 0.8\,y_3 &= y_3
\end{aligned}
$$

which can be rewritten

$$
\begin{aligned}
-0.5\,y_1 \qquad\quad &+ 0.2\,y_3 = 0\\
0.2\,y_1 - 0.4\,y_2 \qquad &= 0\\
0.3\,y_1 + 0.4\,y_2 - 0.2\,y_3 &= 0\,.
\end{aligned}
$$

The solution vector is

$$
\bar{y} = (y_1, 0.5\,y_1, 2.5\,y_1)\,,
$$

where y_1 may be any arbitrary positive real number. In general, the equilibrium income vector will always take the form

$$
\bar{y} = (y_i, b_i y_i, \cdots b_{n-1} y_i) \qquad (4)
$$

for any indecomposable model with n regions

Two tentative conclusions can be derived at this point. In the first place it becomes immediately apparent in the static equilibrium model that if the elements of A are held constant no improvement in a region's relative share of the total available income is possible. Therefore, it follows that any increase in the absolute level of income for a poor region, say Region 2 in the numerical example, must be accompanied by an even greater absolute increase in the incomes of its wealthier trading partners if the exchange model is to remain in equilibrium. The result is a widening of the absolute income gaps between regions.

Secondly, equilibrium growth can take place in the linear exchange model, the limiting requirement being that the growth rate must be the same for each region. As noted above, however, the basic problem is not solved since depressed regions become even more depressed relative to their wealthier neighbors. We remark that this may not be true in terms of utility, depending upon the elasticities of the marginal-utility-of-income schedules of the regions' inhabitants.

The situation is somewhat less hopeless if it is possible to reduce the regional complex into a set of indecomposable subsets of regions. To consider another numerical example, take a five-region model whose exchange matrix, after suitable reorganization, can be written

$$A^* = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \end{pmatrix}$$

Note that Region 5 neither exports to nor imports from any other region and that subsets $A_1 = \begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{pmatrix}$ and $A_2 = \begin{pmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{pmatrix}$ are irreducible submatrices. Proceeding as before, we find that the equilibrium income vector is

$$\bar{y} = (y_1, 2.5\,y_1, y_3, 4\,y_3, y_5)\,,$$

where y_1, y_3, and y_5 again are any nonnegative real numbers.

The conclusions are now less rigid than before. Clearly, Region 1's income cannot be increased relative to the income of Region 2, but the combined incomes of Regions 1 and 2 can be raised relative to the incomes of the three remaining regions. The same result is apparent in the case of Regions 3 and 4, and Region 5's income, being independent of the income of any other region, can be increased unilaterally without violating the equilibrium condition. Hence through a suitable process of aggregation, regions can be redefined in a manner such that regional growth is possible in a manner consistent with trade equilibrium. This solution, however, merely amounts to sweeping the income discrepancies under the rug and does not eliminate the original problem.

2. DYNAMIC REGIONAL GROWTH

We now consider the dynamic properties of the model. In particular, we shall begin with a trade equilibrium as defined in the previous section and then examine the consequences of some exogenous shock, e.g., either a one period injection of income into a depressed region or an increase of autonomous

investment which increases that region's income for all subsequent periods. As before, we first investigate the situation for an indecomposable trade matrix.

If we wish to find the interregional distribution of income after t time periods, we know that

$$\sum_{j=1}^{n} v_{ij}(t) = 1 \tag{5}$$

and

$$v_{ij}(t+1) = \sum_{j=1}^{n} v_{ij}(t) a_{ij} , \tag{6}$$

where the number in parentheses dates the associated variable, and the v_{ij} denote the elements of the ith row, in A^t. Hence the row elements still sum to unity, and Equation 6 introduces a dynamic situation in that a region's income in any time period is given by the total of its sales to all other regions and itself during the previous periods. It follows that the income distribution vector in any period is given by

$$y(1) = y(0)A$$
$$y(2) = y(1)A = y(0)A^2$$
$$y(3) = y(2)A = y(0)A^3$$

and, in general ,

$$y(t) = y(t-1)A = y(0)A^t . \tag{7}$$

A few definitions are in order at this point.

1. A state, say i, is called *periodic* with period h if a return to i is possible in gh steps, where g is a positive integer and $h > 1$ is the largest integer with this property. Thus $a_{ii}^{(t)} = 0$ where t is not divisible by h. A state which is not periodic is called *aperiodic.*

2. A state i is called *recurrent* or *nontransient* if a return to that state is certain. If the probability of a return is lesss than unity, the state is said to be *transient* or *nonrecurrent.* If the expected recurrence time for i is infinite, the state is said to be *recurrent null.* If the expected recurrence time is less than infinite, the state is called *positive.*

3. A recurrent state i which is neither null nor periodic is called *ergodic.*

4. An income vector $\bar{y}$ will be called an *equilibrium income vector* if it satisfies the property

$$\bar{y} = \bar{y}A . \tag{8}$$

The corresponding matrix A will be called *stable.*

We find it extremely convenient in the initial period to divide each region's income by the summed income of all the n regions so that

$$\sum_{i=1}^{n} y_i = 1 \tag{9}$$

for the interregional distribution of income in period 1.

We next state without proofs two basic limit theorems of the theory of discrete Markov chains with a denumerable number of states, and then consider their implications for regional economic development. The proofs are to be

found in Bharucha-Reid [2].

Theorem 1: If the state j is either transient or recurrent null, then

$$\lim_{t\to\infty} a_{ij}^{(t)} = 0 \tag{10}$$

for all i, where $A^t = (a_{ij}^{(t)})$.

Theorem 2: If the exchange matrix A is ergodic, then for every pair of states i and j

$$\lim_{t\to\infty} a_{ij}^{(t)} = vj\,, \tag{11}$$

independent of i, and the v_j satisfy the equation

$$v_j = \sum_{i=1}^{n} v_i a_{ij} \tag{12}$$

as well as Equation 4. Moreover, the v_j are unique.

The importance of these two limit theorems for Markov chains and their implications for the convergence of regional incomes to an equilibrium is obvious. The theorems assert that if all of the states of A are aperiodic and A is indecomposable then all of the elements of A^t converge toward limiting values as t approaches infinity. Another theorem follows, therefore, that the elements of $y(t) = y(0)A^t$ also converge toward limiting values as t increases without bound.

3. APERIODIC CHAINS

For example, consider a simple model consisting of two regions R_1 and R_2 with the exchange matrix

$$A = \begin{pmatrix} p & 1-p \\ q & 1-q \end{pmatrix}$$

where p and q lie between zero and one. Using the method of generating functions, we calculate A^t to be

$$A^t = \frac{1^t}{1-p+q}\begin{pmatrix} q & 1-p \\ q & 1-p \end{pmatrix} + \frac{(p-q)^t}{1-p+q}\begin{pmatrix} 1-p & p-1 \\ -q & q \end{pmatrix}.$$

Since $|p-q| < 1$,

$$\lim_{t\to\infty} A^t = \begin{pmatrix} \dfrac{q}{1-p+q} & \dfrac{1-p}{1-p+q} \\ \dfrac{q}{1-p+q} & \dfrac{1-p}{1-p+q} \end{pmatrix}$$

Clearly, by Equation 7,

$$\bar{y} = y_1, \frac{(1-p)}{q} y_1 \tag{13}$$

A discussion of generating functions and their use is provided in Howard [4] and Saaty [5].

Suppose now that income is increased for one period only in R_1 by an amount to x. The interregional income distribution converges over time to a

new equilibrium given by

$$\bar{y}' = \left((1+x)y_1, \frac{(1-p)(1+x)}{q}y_1\right) \tag{14}$$

An examination of Equation 14 reveals that for the income of Region 1 to increase more than that of Region 2, it is necessary and sufficient that $q > 1 - p$, i.e., that Region 1 spends a greater proportion of its income at home than does Region 2. But from Equation 13 this would make Region 2 the initially depressed area so that the result is same as for the static model.

A simple numerical example is given by

$$A = \begin{pmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{pmatrix}$$

The equilibrium income vector is

$$\bar{y} = (y_1, 1.25\, y_1)\,.$$

Since we want total income to equal unity,

$$\bar{y} = (\tfrac{4}{9}, \tfrac{5}{9})\,.$$

In order to illustrate theorem, we assume a transfer from Region 2 to 1 in the initial period so that

$$y(0) = (0.5,\ 0.5)$$

and examine the behavior of $y(n)$ as t increases indefinitely.

$$\lim_{t\to\infty} A^t = \begin{pmatrix} \frac{4}{9} & \frac{5}{9} \\ \frac{4}{9} & \frac{5}{9} \end{pmatrix},$$

and, therefore,

$$\lim_{t\to\infty} y(t) = \bar{y} = (0.5 \quad 0.5)\begin{pmatrix} \frac{4}{9} & \frac{5}{9} \\ \frac{4}{9} & \frac{5}{9} \end{pmatrix}$$

$$= (\tfrac{4}{9},\ \tfrac{5}{9})$$

If income is raised by x in period 0 in Region 1 we get

$$y(0) = (\tfrac{1}{2} + x,\ \tfrac{1}{2})\,,$$

and

$$\bar{y} = (\tfrac{4}{9}(1+x),\ \tfrac{5}{9}(1+x))\,.$$

Hence y converges to a unique equilibrium with the same relative interregional distribution as formerly.

If $q = 0$, then Region 2 spends all of its income at home, and Region 1 is a transient state such that the equilibrium income vector is

$$\bar{y} = (0,\ 1)\,,$$

and all of the benefits of any one-shot injections will have accrued to Region 2 once the new equilibrium is attained.[2]

However, if the aid to the depressed region is in the form of capital in-

[2] In a typical Markov process Region 1 would clearly be a transient state inasmuch as its limiting state probability is zero. Furthermore, Region 2 is called a *trapping* or *absorbing* state since $a_{22} = 1$, i.e., all income ends up in R_2.

vestment such that the area's output and income are increased during the current and all subsequent periods, the income gap will be partially closed, the amount of the change depending upon the amount of the initial investment and the capital-output ratio. Thus, if model is in equilibrium in period 0 and income in Region 1 is increased in periods $1, 2, \cdots, n$ by an amount equal to x,

$$y(0) = \left(y_1, \frac{(1-p)}{q} y_1\right)$$

$$y(1) = \left((y_1 + x), \frac{(1-p)}{q} y_1\right)$$

$$y(2) = (y_1(1)A + (x, 0)) = \left[((1+x)y_1 + x), \left(\frac{(1-p)}{q}(y_1 + x)\right)\right]$$

$$y(3) = (y_1(2)A + (x, 0)) = \left[((1+2x)y_1 + x), \left(\frac{(1-p)}{q}(y_1 + 2x)\right)\right]$$

.....

We conclude that in the case of aperiodic states and indecomposable matrices it is possible to raise the relative incomes of depressed regions through continual and constant injections by an amount equal to the the injection.[3]

4. PERIODIC CHAINS

In the case of periodic chains the components of A^t do not converge toward limiting values as n becomes very large but tend to oscillate indefinitely, resulting in an oscillation in the componets of the regional income distribution vector. These oscillations, however, are predictable. As defined on paper 12, a periodic chain is a recurrent chain such that for every fixed state the chain will occupy that state with probability equal to one after h, $2h$, $3h$, $\cdots$ transitions, where h is an integer denoting the periodicity of the chain.

Consider again the simple two-regional model with $p = 0$, and $q = 1$, i.e.,

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

That is, Region 1 spends all of its income in Region 2 and vice versa. We have

$$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In general, using an appropriate generating function,

$$A^t = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} + (-1)^n \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix},$$

where $h = 2$.

[3] We implicitly assume that no opportunity costs are connected with the initial investment. Hence an increase in the capital stock of depressed regions is not accompanied by a reduction in the income base of other regions.

Hence the system does not move toward an equilibrium in the sense that the $a_{ij}^{(t)}$ converge to any limiting values. If the inital income vector is $y(0) = (y_1, 1 - y_1)$, we have

$$y(1) = y(0)A = ((1 - y_1),\ y_1)$$
$$y(2) = y(0)A^2 = (y_1,\ (1 - y_1))$$
$$y(3) = y(0)A^3 = ((1 - y_1),\ y_1)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

and the interregional distribution of income fluctuates indefinitely for any $y_1 \neq 0.5$.[4]

Suppose next that a one-shot attempt is made to increase income in Region 1. We see that

$$y(0) = ((y_1 + x),\ (1 - y_1))$$
$$y(1) = y(0)A = ((1 - y_1),\ (y_1 + x))$$
$$y(2) = y(0)A^2 = ((y_1 + x),\ (1 - y_1))$$

Once more we note that while Region 1 is better off in an absolute sense it has not gained relatively. Moreover, the amplitude of the oscillation has increased.

If, on the other hand, the injection x is continued indefinitely we have

$$y(0) = ((y_1 + x),\ (1 - y_1))$$
$$y(1) = y(0)A + (x,\ 0) = ((1 - y_1 + x),\ (y_1 + x))$$
$$y(2) = y(0)A^2 + (x,\ 0) = ((y_1 + 2x),\ (1 - y_1 + x))$$

We conclude once more, therefore, that the only way a depressed region's income can be increased relative to the income of the other regions of an irreducible exchage matrix, periodic or not, is via either a capital investment program or some other program designed to maintain a continuous flow of new income in the low-income region.

5. DECOMPOSABLE CHAINS

We now consider the exchange matrix

$$A = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{pmatrix}$$

The indecomposable submatrices A_1, A_2, A_3, and A_4 are all familiar to us by this time. The equilibrium income vector is

$$\bar{y} = (y_1, 1.25\,y_1, 0, y_2, y_3, y_3, y_4)\,.^5$$

[4] If Equations 6 and 7 are used to find an equilibrium income vector, $\bar{y} = (0.5,\ 0.5)$, in which case both regions would have stable income over time. However, $y(n)$ does not converge to these values as $n \to \infty$.

[5] See footnote 4 with respect to the solution of A_3.

As in the static case a single injection will increase the income of all the regions in any indecomposable submatrice relative to the incomes of other regions. However, in order to raise the income of any single region which is a proper subset of one of the irreducible submatrices, relative to the incomes of all other regions, it is necessary to make the stream of new income in that region continuous. Otherwise, the equilibrium requirements would force relative incomes within each submatrix back to their former relationship.

6. CONCLUSIONS

In summary, if the economic development of a region is defined as an increase in that region's income relative to the income of other regions, such growth is consistent with trade equilibrium, and hence possible, only under special circumstances. In the first place, the relative incomes of an indecomposable trading bloc of regions can be increased by a once-and-for all injection. However, the income of a region or regions within such a bloc can be raised relative to the incomes of other regions within the bloc only by continuous injections, hence necessitating either a continuous flow of payments into the depressed region or capital outlays designed to enlarge the region's income base.

We have assumed throughout the propensities to spend remain constant. Buy-at-home campaigns and import duties, where possible and in the absence of retaliation, can change the components of a trade matrix in a manner favorable to the depressed region. Moreover, beneficial adjustments in the a_{ij} are apt to take place if the capital projects are designed to produce exported or import-competing commodities, especially if the region enjoys a comparative advantage in those areas. This points out the principal difficulty of such a program; the need for retraining the labor force exists, whether the solution entaills relocating the workers in other regions or the development of new industries in the depressed region.

REFERENCES

[1] Baumol, W. J., *Econcomic Theory and Operations Analysis*, Englewood Cliffs: Prentice-Hall, 1961, Chapter 16.

[2] Bharucha-Reid, A. T., *Elements of the Theory of Markov Processes and Their Applications*, New York: McGraw-Hill, 1960, pp. 28–30 and Appendix A.

[3] Gale, D., *The Theory of Linear Economic Models*, New York: McGraw-Hill, 1960, Chapter 8.

[4] Howard, R. A., *Dynamic Programming and Markov Processes*, New York: Wiley, 1960, pp. 7–12.

[5] Saaty, T. L., *Mathematical Methods of Operations Research*, New York: McGraw-Hill, 1959, pp. 77–84.

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AN APPROACH TO THE ANALYSIS OF METROPOLITAN RESIDENTIAL EXTENSION†

by Willard B. Hansen*

1. INTRODUCTION

This paper summarizes the results of an analysis of metropolitan residential extension, defined here as "the new occupancy of intrametropolitian open sites by urban housing during specified time periods." Extension is viewed in terms of the residential development levels of metropolitan subregions. A series of models which yield subregional residential development levels are described, and the efficiency of these models is investigated by the regression method using data for the Philadelphia Metropolitan Region, during the periods 1940–50 and 1950–56.

Metropolitan residential extension may be regarded as one aspect of the more general phenomenon of urban residential location. There are several previous instances of the formulation and empirical testing of urban residential location models.[1] These prior models are alike in a number ways. They are predictive rather than prescriptive, define the entire urban area as their spatial setting, and yield levels of residence for urban subregions (with subregional residence levels being expressed as static distributions at a given point in time, or as distributional change over time). The models are based mainly on empirical observation, exhibit simple algebraic forms, and describe presumed relationships among aggregates, rather than actual behavioral processes. All embody single equations rather than simultaneous equation systems, and contain parameters whose numerical values must be estimated empirically. Finally, application of the models to projection requires the use of iterative adjustment mechanisms for bringing the output values into conformance with expected subregional residential capacities, and with the residence level anticipated for the urban area as a whole.[2]

Although the models which constitute the subject of this study are generally similar to the ones cited above, they differ from the latter in one important respect. Whereas their precursors treat subregional residence levels as a static

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1. For example, the work of Blumenfeld [2], Bogue and Harris [3], Clark [6], Hamburg and Creighton [11], Walter G. Hansen [12], Schmitt [16] and Thomas [18].

2. The recent theoretical work of Alonso [1] and of Herbert and Stevens [13] has resulted in relatively complex "behavioral" models of residential location. To the writer's knowledge, these models have so far not been applied empirically.

or changing distribution of housing or residents, the models of the present study define such levels in terms of open site dwelling construction, i.e., of residential extension. In the writer's view, this distinguishing feature constitutes an important improvement, an improvement that follows from the relatively greater homogeneity characterizing the determinants of residential extension, and from the strategic role of residential extension in urban locational change.

The homogeneity characterizing the determinants of residential extension in United States metropolitan regions during the 20th Century results from the fact that extension has taken place largely in the form of dwelling construction by private homebuilders and that it has been subject to comparatively few (although steadily increasing) public controls. In contrast, the determinants of population or dwelling distribution are much less homogeneous. Thus, change in the dwelling levels of urban subregions may be the result of any or all of the following:

1. Construction of new dwellings on open sites
2. Construction of new dwellings on cleared sites
3. Conversion of existing structures from nonresidential to residential use, or vice versa
4. Creation of new dwellings through the partitioning of existing ones
5. Elimination of dwellings through the merging of two or more units
6. Demolition of dwellings

Subregional population change, in addition to being subject to the six types of action listed above, may also comprise births and deaths, in- and out-migration, and growth and decline in non-household population (rooming house, college, hospital, military etc.).

The strategic role of residential extension in the locational change occurring in U. S. metropolitan regions is evident in several ways. Extension has accounted for most of the locational change marking metropolitan residence. In its turn, the locational pattern of metropolitan residential extension has determined the distribution of new public and private facilities directly serving the occupants of new housing: e.g. facilities such as stores, schools, parks, water and sewer mains, streets etc. Finally, new housing has been the single most important consumer of open sites, while a large share of the remaining open-site consumption has been made up of public and private facilities directly serving the occupants of such new housing.[3]

2. MODELS OF SUBREGIONAL RESIDENTIAL DEVELOPMENT

A. *General Statement*

A general model of subregional residential development may be written as:

$$Y_i = \phi(_iX_i) ,$$

3. The relative importance of residental development is illustrated by the growth pattern exhibited by the northeastern portion of Philadelphia County Between 1944 and 1954. The area in question comprises 44 square miles, of which fewer than half were in urban use in 1944. Residential uses accounted for 84% of the 4.8 square mile increase in developed land that took place in the area during the 1944-54 period. See Philadelphia City Planning Commission [15], Table 7, Sections K and L.

where: Y_i = level of residential development in subregion i
$_kX_i$ = level of residential development factor k in subregion i
ϕ = development function prevailing in the metropolitan region during the development period
$i = 1 \cdots n$ = set of residential development subregions
$k = 1 \cdots m$ = set of residential development factors.

The individual terms of the general model are defined as follows:

Metropolitan Region

A relatively large urban agglomeration containing (1) one or more extensive and densely developed cores, and (2) a sizable fringe area which has lower overall settlement density than the core(s), and the individual parts of which bear a close socioeconomic relationship with, or are within commuting distance of, at least one core. This concept is similar to that used by the U. S. Bureau of the Budget [19].

Residential Development Period

A period 2 through 10 years in length, for which residential development rates are predicted.

Development Subregion Set

A set of areas forming part of one given metropolitan region; where each area is bounded and continuous, contains a substantial quantity of open sites available for residential construction during a specified development period, and includes little or no urban housing.

Residential Development

The gross or net change in the absolute or relative number of a given category of residences, that takes place in a development subregion during a development period. The term "residence" refers to an individual unit of shelter, or a set of persons occupying such a unit. Three development rates considered are dwelling construction volumes, dwelling capacity utilization ratios, and incremental population densities.

Residential Development Factor

An independent variable which is used to explain variations in subregional residential development levels. Four development factors are considered: Residential Settlement, Centrality, Residence Access and Employment Access.

Development Function

An algebraic statement which relates residential development levels to development factor levels. Three development function types are considered: linear, exponential and power.

The dependent variable of the general model expresses metropolitan residential extension in terms of subregional residential development levels, i.e., of change over time in the absolute or relative number of specified types of residences in metropolitan development subregions. The dependent variable was given this broad formulation to enable it to subsume a set of alternative development rates which included both direct and indirect measures of residential extension (see Section C below).

Two to ten years appeared to be a reasonable range for the time periods during which the development factors under consideration might be expected to influence development rates "unilaterally". Where the period exceeds 10 years,

"multilateral" relationships requiring simultaneous equations would be likely. Where the period is less than two years long, special local circumstances might be expected to distort the relationships.

B. *Development Function Types*

Three development function types are considered and may be expressed algebraically as follows:

1. Linear Function Type

$$Y = a_0 + a_1(X_1) + \cdots + a_m(X_m)$$

2. Exponential Function Type

$$Y = (a_0)(a_1)^{X_1} \cdots (a_m)^{X_m}$$

3. Power Function Type

$$Y = (a_0)(x_1)^{a_1} \cdots (X_m)^{a_m}$$

where Y is the dependent variable; $X_1 \cdots X_m$ are the levels of independent variables $1 \cdots m$; a is a base parameter; and $a_0 \cdots a_m$, are independent parameters associated with variables $1 \cdots m$.

All three function types are "monotonic" in the sense that the value of the dependent variable will either rise with every increase in a given independent variable, or fall with every such increase, all other variables being held constant. The monotonic property of the forms is in accord with current theory about the nature of the relationships between residential development levels and the individual development factors that were investigated; the relationships in question may be expected to be either consistently positive or consistently negative. At the same time, the small number of parameters ($M+1$ for any combination of M variables) makes the three function types particularly suitable for regression analysis. Where regression coefficients are interpreted as sample parameters from a "hypothetical universe" the explanatory ability and parametric reliability of the regression equations varies directly with the number of degrees of freedom entailed in their formulation.[4]

C. *Development Rates*

Three alternative expressions of development (development rates) are discussed below. Each expression specifies the general term of subregional residential development, which may in turn be regarded as one way of describing the phenomenon of urban residential extension.

1. *Dwelling Construction Volume* (Q)

$$Q = H_i$$

2. *Dwelling Capacity Utilization Ratio* (V)

4. The linear function type is incorporated in the residential location models of Blumenfeld [2], Bogue and Harris [3], Schmitt [16] and Thomas [18]. The exponential form is employed by Hamburg and Creighton [11], while the power form is employed by Walter G. Hansen [12]. In every one of these instances, the forms are employed to specify relationships which are analogous to the development function of the present study, although there are, of course, material differences in the interpretation of the variables involved.

$$V = \frac{H_i}{(J_i)(L_i)}$$

3. *Incremental Population Density* (Z)

$$Z = \frac{U_i - P_i}{A_i}$$

where: H_i = number of nonfarm dwellings added for sale or rental in development subregion i during a development period, by commercial or ownerbuilders; on privately acquired open sites, and not as part of a comprehensively designed and executed mass housing project.[5]

L_i = amount of open site area available in subregion i, during a given period, for the addition of new private nonfarm dwellings (H_i); where the term "open site area" denotes land which is vacant or in non-urban use

J_i = mean number of new private nonfarm dwellings (H_i) that can economically be constructed during a given period per unit of open site area (L_i)

P_i = number of persons residing in subregion i at the beginning of a given period

U_i = number of persons residing in subregion i at the end of the period

A_i = total area of subregion i

Development Rates Q and V both constitute direct expressions of the phenomenon of urban residential extension. The former treats dwelling construction volumes in absolute terms, while the latter standardizes these volumes for subregional dwelling capacity.

Development Rate Z constitutes an indirect expression of residential extension. Its formulation was motivated by the need for a measure that could be applied in the Philadelphia Metropolitan Region for an interval earlier than 1950–56 as well as for that period itself (the data required by Rates Q and V were not obtainable for the Region previous to 1950). Rate Z appeared to be suitable for this purpose because population and total area data were available for both the 1940–50 and 1950–55 periods. Moreover, the distribution of Rate Z rather closely resembled the distribution of the "direct" rate, V. This correspondence is reflected by the high degree of association ($\bar{r} = +\cdot 83$) which was registered in the Region for the 1950–55/56 period between Rates V and Z. The correspondence stemmed from the criterion which required the development subregions to contain substantial amounts of open site area, while not permitting them to include sizable quantities of old urban housing. This requirement led to a situation in which subregional population increments were largely brought about by the occupancy of new private dwellings constructed on open sites (as demonstrated by an $\bar{r}$ of $+\cdot 89$ between 1950–55 population increments and 1950–56 open site dwelling construction volumes). For the same period, the "total area" which is incorporated in Rate Z as a measure of subregional dwelling capacity was also markedly associated with the capacity measure, (J_i)(L_i), incorporated in Rate V ($\bar{r} = +\cdot 67$). It seems reasonable to assume that

5. It should be noted that the term "dwelling", as used here, includes units in new detached houses as well as well as units in new row houses, apartment buildings, etc.

the close correspondence observed in the Philadelphia Metropolitan Region during 1950–55/56 between Rates V and Z prevailed there during 1940–50 as well.[6]

D. *Development Factors*

Four development factors are considered: Residential Settlement, Centrality, Residence Access and Employment Access.

The *Residential Settlement* variable expresses the degree to which urban housing and local community facilities are present in a given development subregion at the beginning of a particular period. Subregional residential development levels may be presumed to vary directly and substantially with subregional levels of residential settlement.[7] This assumption stems from the types of facilities (e.g., streets, schools, stores, water supply and sewerage systems) which are commonly associated with residential settlement in metropolitan regions. Availability of facilities of the indicated type acts to induce private dwelling construction in one or both of two important ways: by lowering site preparation and building costs, and by providing community services which will tend to make new dwellings more attractive to prospective customers.

The *Centrality* variable expresses the proximity of the development subregion to the dominant intrametropolitan center. Subregional residential development levels may be presumed to vary with centrality because of the opportunities which the dominant center offers prospective dwelling customers by virtue of the fact that it constitutes a transportation hub and a center of government, business and cultural activity.

The *Residence Access* variable expresses degree of subregional proximity to all metropolitan residents at the beginning of given period. Development levels may be presumed to vary with residence access levels because the distribution of residents within the metropolitan region as a whole defines any given home-builder's market population of dwelling customers and the social opportunities that exist for those customers themselves. Proximity to metropolitan residents thus acts to induce private dwelling construction in two ways: by expediting the disposition of new dwellings, and by yielding transportation advantages which will tend to make the new dwellings more attractive to prospective customers.[8]

The *Employment Access* variable expresses degree of subregional proximity to all metropolitan jobs during a given period. Development levels may be presumed to vary with employment access levels because the intrametropolitan distribution of jobs delimits the employment opportunities of dwelling customers, and length of work-trip is a major factor in the decision to buy or rent a new

6. The correspondence in question may be expected to hold also for other periods and metropolitan regions. Data on the population and total area of small territorial units are obtainable for some of the earliest decennial U. S. Census years. Rate Z therefore suggests itself as a useful "proxy" variable for the comparative analysis of development rates which constitute direct expression of urban residential extension.

7. Note that the concepts of *residential settlement* and *residential development* refer, respectively to amount of residence at a given point in time, and to change in amount of residence over time.

8. Strictly speaking, the residence access variable expresses two kinds of proximity: *accessibility* with respect to a dispersed market population of dwelling customers, and *access* with respect to the dispersed social opportunities existing for those customers.

dwelling.

The four development factors under consideration may be presumed to act in a largely "unilateral" manner on development levels during middle range (2 through 10 year) periods. It should be obvious that the four factors will tend to influence one another, and that still other factors will tend to influence development.

The four development factors are defined as:

1. *Residential Settlement* (B)

$$B = \frac{P_i}{A_i}$$

2. *Centrality* (C)

$$C = D_{ic}$$

3. *Residence Access* (M)

$$M = \sum_{j=1}^{n} P_j(2.5 + D_{ij})^{\theta} \qquad \theta \neq 0;\ 2.5 + P_{ij} \leqq 35$$

4. *Employment Access* (E)

$$E = \sum_{j=1}^{n} T_j(2.5 + D_{ij})^{\theta} \qquad \theta \neq 0;\ 2.5 + D_{ij} \leqq 35$$

where: $i = 1 \cdots n$ = set of residential development subregions

$j = 1 \cdots n$ = set of opportunity subregions; where the elements of the set together make up the metropolitan region containing development subregions $1 \cdots n$, and include subregions $1 \cdots n$

A_i = total area of subregion i

P_i(or P_j) = number of persons residing in subregion i (or j) at the beginning of a period.

c = intrametropolitan center

D_{ij}(or D_{ic}) = number of airline miles between the centroid of subregion i and the centroid of subregion j (intrametropolitan center c); where centroids represent the points which are most accessible to the population residing in a subregion or center during the period

T_j = number of jobs in subregion j during the period

2.5 = terminal airline distance constant prevailing in the metropolitan region

35 = airline distance limit prevailing in the metropolitan region

θ = airline distance exponent

Population levels used in the residential settlement and residence access variables refer specifically to the beginning of the period, whereas the job levels in the employment access variable may refer to any time during the period. The condition imposed on population levels obviates the circularity which would otherwise follow from the fact that the course of residential development itself influences population distribution during any given period.

The terminal distance constant of 2.5 miles corresponds to 5 minutes driving time at an average speed of 30 miles per hour. This constant reflects the

minimum movement effort required for trips using the available transportation media in U. S. metropolitan regions during the 1940's and 1950's.[9] The mileage limit value of 35 reflects the maximum distance which an individual would be willing to commute. It corresponds to 70 minutes driving time at 30 miles per hour.

The airline distance exponent θ in the residence and employment access factors may take any positive or negative value. Where the exponent is positive the variable may be regarded as a measure of "travel cost". Where the exponent is negative the variable may be regarded as a measure of "potential".[10]

3. EMPIRICAL TESTS OF THE MODELS

Empirical tests were applied to a series of models incorporating alternative combinations of the function types, development rates and development factors discussed above. The efficacy of these models was investigated by the regression method using data for the Philadelphia Metropolitan Region and the periods 1940–50 and 1950–56.

A. *The Philadelphia Metropolitan Region*

In addition to meeting practical requirements of data availability, the particular case selected for the statistical tests was satisfactory from a substantive point of view. Between 1939 and 1957 the Philadelphia Region experienced considerable population growth, most of which was accommodated through residential extension. Moreover, with regard to volume, physical and financial characteristics, and type of entrepeneurial organization, homebuilding in the region generally followed the pattern prevailing in U. S. metropolitan regions as a whole during the period under consideration.

The Philadelphia Metropolitan Region was defined as a 4565 square mile area centering on the City of Philadelphia, and having a total population of 4.2 million in 1950. The Region is composed of 11 counties situated in the States of Pennsylvania, New Jersey and Delaware, and comprises the Philadelphia, Trenton and Wilmington Standard Metropolitan Statistical Areas (Figure 1). The central district ("downtown") of Philadelphia is the only dominant center. Generally speaking, topography and soil conditions favor urban development.

Between 1939 and 1957, the population of the Philadelphia Metropolitan

9. It may be noted that the "potential" variables of Walter C. Hansen's [12] residential location models, in which movement effort is expressed as driving time, incorporate a terminal constant of 5-6 minutes.

10. For a discussion of the "potential" concept, see Carrothers [4]. for a comparison of the "potential" and "travel cost" concepts, see Dunn [8].

Indices of aggregate access have been employed as independent variables in location models generally, and in urban residential location models in particular. While access has been expressed as "potential" in nearly all of these cases, there are also some references to the use of "travel cost". Duncan [7] has asserted that the "potential" and "travel cost" concepts are closely related. This view is rejected by Stewart and Warntz [17], who maintain that the "obvious mathematical similarity is merely formal",and who contrast what they call the "sterility" of "travel cost" with the "manifold applications" of "potential".

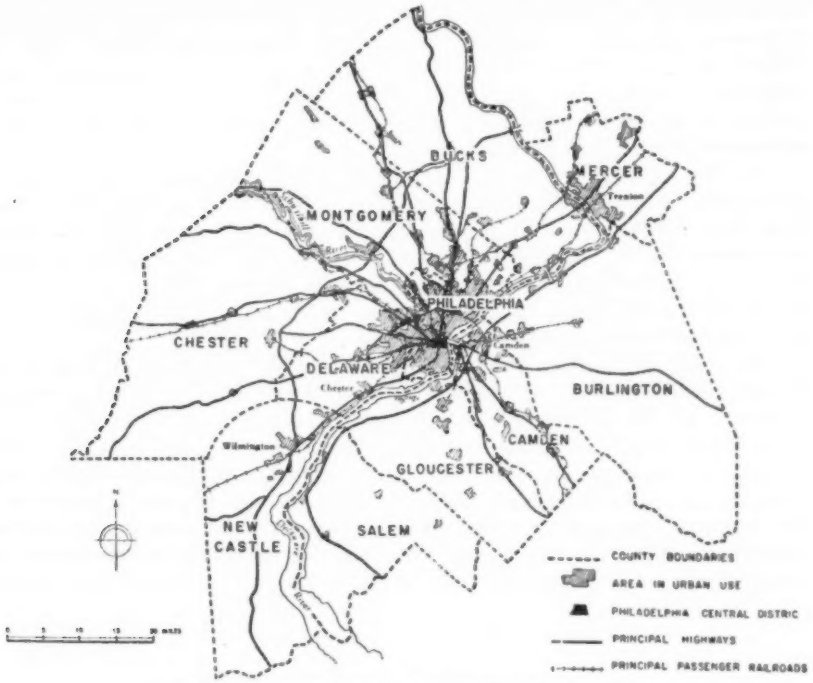

FIGURE 1: Philadelphia Metropolitan Region, 1950

Region increased by one third, from 3.6 million to 4.8 million. During that same period, approximately 370,000 new dwellings were constructed in the eight-county Philadelphia Standard Metropolitan Statistical Area. Together, they accounted for 88.7% of all additions to the stock of dwelling units. Less than 1% represented farm dwellings, less than 1% were built as public housing, and most were constructed on open land.

During the 1940–50 period, the large majority of new private dwellings were started by small scale commercial builders. In general, this entrepreneurial pattern continued through the 1950–56 period, despite an increase in the number of large scale builders. In addition, however, the latter period was marked by the construction of two private mass housing projects, Levittown and Fairless Hills in the lower part of Bucks County. They contained about 15,000 and 5,000 dwellings respectively.[11]

B. *Statistical Procedures*

Several hundred bivariate and multivariate regression equations were formulated, and their parameters were estimated by least squares. Each equation yielded subregional development levels as a function of subregional levels of

11. For data on residential construction in the Philadelphia and other metropolitan regions, see U. S. Census [20] and U. S. Bureau of Labor Statistics [21].

one or more of the development factors under consideration: i.e., Residential Settlement, Centrality, Residence Access and Employment Access. The individual equations differed by function type, subregion set, development rate, development factor index, and factor combination. Calculation of the regression coefficients, efficiency measures and access indices ("potential" and "travel cost") necessitated the use of electronic computers.

The development subregion sets selected for study were situated in the Philadelphia Standard Metropolitan Statistical Area. They excluded all portions of the Area which were predominantly in urban use at the beginning of a given development period. Moreover, the sets used for the 1950–56 period excluded the portion of Bucks county containing the mass housing projects of Levittown and Fairless Hills. Most of the equations were based on 44 subregions, with the remainder employing 47 or 71 subregions.

The metropolitan subregions used in the analysis were selected from a set of areal subdivisions, called Planning Analysis Areas, that had been previously delineated by the Philadelphia City Planning Commission and the Philadelphia Urban Traffic and Transportation Board. The Planning Analysis Areas were adopted by the writer because they constituted the only set of suitably sized subregions for which data were available. The subregions varied considerably with respect to shape and size, being relatively small in the densely settled central portion of the Region, and becoming progressively larger toward the fringe. For example, Area No. 2 in Delaware County contained 10.3 square

FIGURE 2: Philadelphia Metropolitan Region Planning Analysis Areas

miles, whereas Area No. 7 in Burlington County contained 377.7 square miles (see Figure 2).

Historical data on the dependent and independent variables were obtained from the U. S. Census and from various local agencies. In some instances, reliability of the local agency estimates was relatively low.

The efficiency of the regression equations was appraised in terms of their explanatory precision and completeness,[12] and parametric reliability and plausibility.[13] The following measures and tests were applied:[14]

1. Explanatory Precision was expressed by corrected residual error ratios ($\bar{H}$) for linear equations,[15] and by corrected residual errors ($\bar{S}$) for exponential and power equations.
2. Explanatory completeness was expressed by corrected coefficients of bivariate and multivariate determination ($\bar{r}^2$ and $\bar{R}^2$ respectively).
3. Parametric Reliability was expressed by cofficient error ratios ($_sC$).[16]
4. Parametric Plausibility was tested by the conformance of the independent regression coefficient signs to generally accepted theory.

C. *Overall Efficiency*

The efficiency of the regression equations varied greatly. Levels of explanatory precision ranged from $\bar{H}$ values of 1.82 to 0.62 in the linear cases, and from antilog $\bar{S}$ values of 4.02 to 1.76 in the exponential and power cases. Levels of explanatory completeness ($\bar{r}^2$, $\bar{R}^2$) ranged from .000 to .875. The coefficient error ratios ($_sC$) of nearly all bivariate regression coefficients were significant at the .01 normal probability level, and their signs conformed to generally accepted theory in every instance. By contrast, a number of the multivariate coefficient error ratios were not significant at even the .05 probability level, and the signs of many multivariate coefficients failed to conform to accepted theory.

By and large, the differentials in efficiency were consistent between the 1940–50 and 1950–56 periods.

Degree of subregional aggregation affected the efficiency of the regression equations in varying ways. While explanatory precision and completeness tended to vary directly with aggregation, parametric reliability varied inversely with it. Table 1 provides a comparison of results obtained for a "basic" set of 44 development subregions and a "disaggregated" set of 71 subregions (the two sets under consideration comprised an identical 3439 square mile section

12. The distinction between explanatory precision and completeness is suggested in Ezekiel [9], p. 159.

13. The first three of these measures must by applied for meaningful comparisons of regression equations which are based on different dependent variable distributions. Investigation of the third and fourth measure is required for all multivariate equations

14. The measures of explanatory precision and completeness are corrected for the degrees of freedom entailed in the calculation of the regression coefficients.

15. Use of the residual error ratio ($\bar{H}$) is suggested in Bogue and Harris [3], p. 30. The ratio is calculated by dividing the residual standard error by the mean of the dependent variable distribution.

16. The coefficient error ratio is computed by dividing the regression coefficient by its standard error. See Ezekiel [9], p. 322.

of the Philadelphia Metropolitan Region, with every one of the 71 subregions of the "disaggregated" set being contained one of the 44 subregions of the "basic" set)

TABLE 1: Efficiency of Bivariate Regression Equations Yielding Development Rate Z as a Linear Function of Residential Settlement (B): 1950-55

	"Basic Set"	"Disaggregated Set"
	Distributional Characteristics	
Number of Subregions	44	71
Average Size of Subregions	78.1 sq. mi.	48.4 sq. mi.
Mean of Dependent Variable Z	244.8	424.8
Coefficient of Variability of Z	1.67	1.91
	Efficiency	
Explanatory Precision ($\bar{H}$)	1.26	1.54
Explanatory Completeness ($\bar{r}^2$)	.44	.36
Parametric Reliability ($_sC$)	5.89	6.38

The explanatory precision of the "basic" equations was moderately superior to the precision of the "disaggregated" equations; the explanatory completeness of the former was still superior, but less so; and their parametric reliability was slightly inferior. Table 1 illustrates this for a pair of equations incorporating Residential Settlement as their independent variable.

Two circumstances help account for the disparate performances of the equations. First, the variability of the dependent variable distribution of the "basic" subregion set was lower than that of the "disaggregated" set.[17] Second, historical data were more reliable in the former case than in the latter, since the "basic" subregions tended to be larger than the "disaggregated" ones.

The slightly inferior performance of the "basic" equations with respect to parametric reliability follows from the algebraic properties of the coefficient error ratio ($_sC$), which act to translate the substantially higher number of observations underlying the "disaggregated" dependent variable distribution into a sizable increase in the $_sC$ levels of the disaggregated equation.[18]

D. *Function Types, Development Rates and Factors*

The efficiency of the power equation forms tended to be moderately to

17. The coefficient of variability is calculated by dividing the standard deviation of a given distribution by its mean. See Hagood and Price [10], p. 126.

18. Several regression equations based on a subregion set including the two subregions containing Levittown and Fairless Hills were formulated. The efficiency of this group was compared with the accuracy of a group of equations incorporating identical variables, and differing only by virtue of being based on a subregion set excluding the two subregions in question. In every case, the $\bar{r}^2$ levels of the former series were moderately lower than those of the latter. As might be expected, observed development rates for the two subregions containing the mass housing projects were substantially higher than the rates estimated by the equations of the former series.

substantially higher than that of the linear and exponential forms, which were similar to each other in overall performance. Exponential equations tended to be better than linear with all the independent variables except the Residential Settlement Factor. Representative results are given in Table 2.[19]

TABLE 2: Levels of $\bar{r}^2$ and $\bar{R}^2$ from Regression Equations Yielding Development Rate V as Alternative Functions of Four Development Factors. 1950-56: N=44

		Linear	Exponential	Power
($\bar{r}^2$)	Residential Settlement (B)	.7042	.3401	.8003
	Centrality (C)	.3728	.6715	.6415
	Residence Access (M_*)	.7229	.7929	.8183
	Employment Access (E_*)	.6090	.8183	.7845
($\bar{R}^2$)	B, C, M_*, E_*	.8023	.7909	.8752

Note: The symbol $_*$ is used in this paper to denote the aggregate access which registered the highest bivariate coefficient of determination with a particular function type and development rate.

The precision of equations yielding the absolute Rate Q tended to be somewhat higher with all three function types than that of equations yielding the relative rates, V and Z. With respect to completeness, on the other hand, Q tended to be poorer than V and Z. (Because of the algebraic properties of the measures involved, parametric reliability changed together with explanatory completeness in bivariate cases, while varying directly with completeness and inversely with degree of intercorrelation among the independent variables in multivariate cases.) Table 3 illustrates this for a series of bivariate equations incorporating Residential Settlement as their independent variable.

Two conditions help account for the finding that the explanatory precision of equations yielding the absolute development rate tended to be generally greater than that of equations yielding the relative rates, despite the greater explanatory completeness attained by the latter. First, the variability characterizing the absolute rate, Q, was substantially lower than that of the relative rates, V and Z. Second, the relative rates — unlike the absolute one — standardize subregional residential development levels for amount of subregional dwelling capacity. As a result, the structural correctness of equations yielding the relative rates may be presumed to be concommitantly greater.

As an example of the distribution of a specific development rate, Figure 3 shows subregional dwelling capacity utilization ratios (V) in the Philadelphia Metropolitan Region during 1950-56. While the density of development generally decreases from the center, the preponderance of development on the Pennsylvania side of the Delaware River should be noted. (The lower excluded area on the map comprises territory which was predominantly in urban use in 1950; the

19. The regression coefficients were calculated by a least squares procedure, which requires logarithmic transformation of the dependent variable in exponential and power cases. See Mills [14], Chapt. 18, and Appendix D of the writer's Ph.D dissertation for a discussion of the comparability of regression results based on, respectively, arithmetic and logarithmically transformed dependent variable distributions.

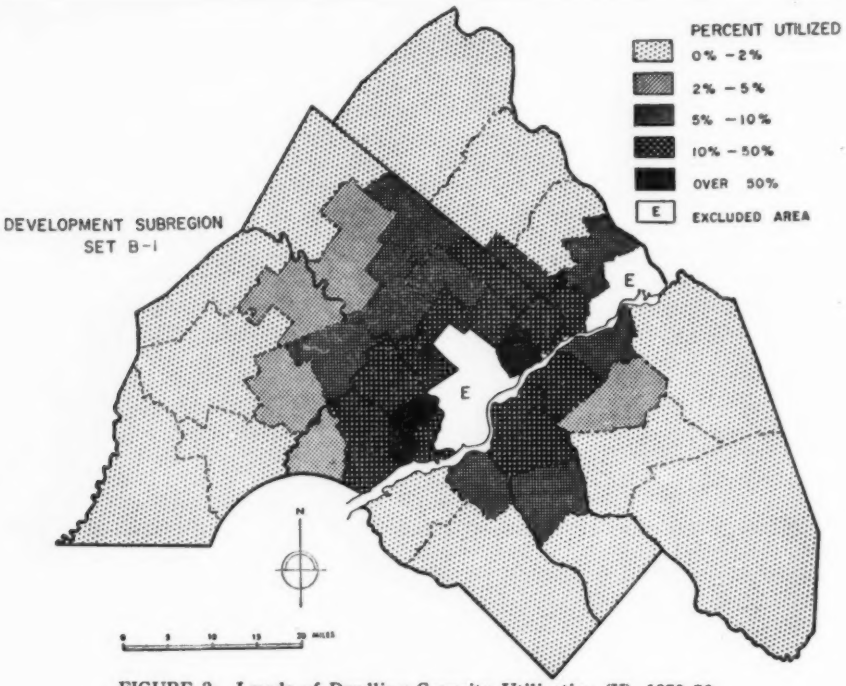

FIGURE 3: Levels of Dwelling Capacity Utilization (V), 1950-56; Philadelphia S.M.S.A.

TABLE 3: Levels of Explanatory Precision and Completeness for Bivariate Regression Equations Yielding Alternative Development Rates as a Function of Residential Settlement (B); 1950-55/56; N = 44

	Development Rates		
	$\frac{Q}{1950\text{-}56}$	$\frac{V}{1950\text{-}56}$	$\frac{Z}{1950\text{-}55}$
Coefficient of Variability	.87	1.38	1.67
Explanatory Precision			
Linear ($\overline{H}$)	.753	.759	1.262
Exponential (antilog $\overline{S}$)	2.47	3.69	4.02
Power (antilog $\overline{S}$)	1.84	2.05	2.53
Explanatory Completeness ($\overline{r^2}$)			
Linear	.2682	.7042	.4414
Exponential	.1363	.3401	.2454
Power	.5344	.8003	.6622

Note: The linear residual errors underlying $\overline{H}$ are additive constants. The antilogs of the exponential and power residual errors are constant ratios.

upper excluded area comprises the territory in which the mass private housing projects of Levittown and Fairless Hills were built between 1950–1956.)

A comparison the efficiency of bivariate regression equations incorporating

TABLE 4: Efficiency of Power Regression Equations Yielding Development Rate V as a Function of One or More Development Factors: 1950-56: N = 44

No. of Indep. Variables	$\bar{r}^2$ or $\bar{R}^2$	Parametric Plausibility (Y or N) and Reliability $_sC$ of the Independent Coeffiicients			
		B	C	M (θ=1.5)	E (θ=2.0)
1	.8003	(Y) 13.05			
1	.6415		(Y) 8.82		
1	.8183			(Y) 13.93	
1	.7845				(Y) 12.96
2	.8533	(Y) 3.32		(Y) 3.90	
3	.8637	(Y) 3.99		(Y) 3.69	(Y) 2.03
4	.8752	(Y) 4.50	(Y) 2.16	(Y) 4.41	(N) 2.58

Notes: 1. The letters Y and N denote, respectively, the plausibility or non-plausibility of the algebraic signs of the regression coefficients.
2. Coefficient error ratios ($_sC$) of 1.96, 2.33 and 2.58 are associated with normal probability levels of, respectively, .05, .02, and .01.

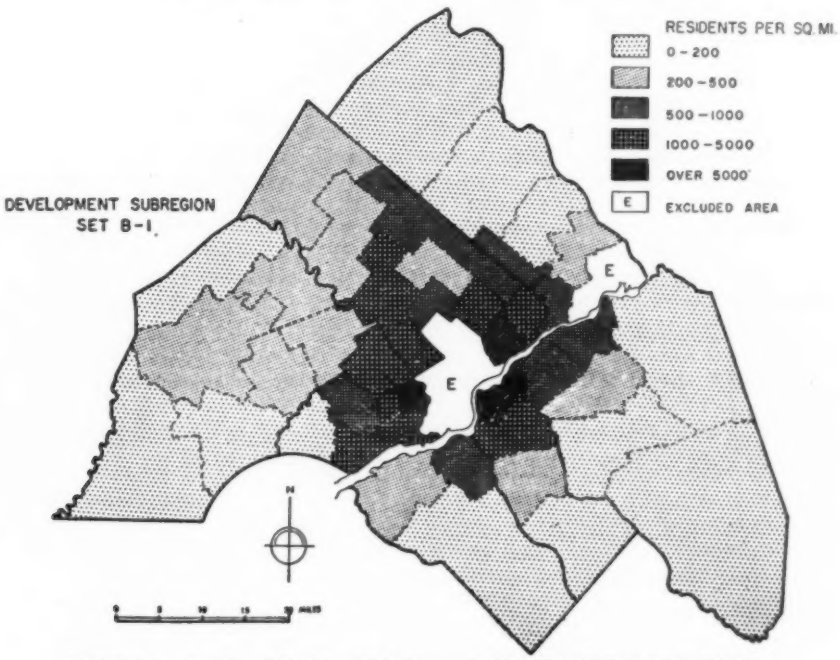

FIGURE 4: Levels of Residential Settlement (B), 1950; Philadelphia S.M.S.A.

the "most explanatory" indices of the four factors revealed differences in performance that were largely consistent by function type. (see Table 2)

The combination of development factors into multivariate regression equations produced varying effects on efficiency. Generally speaking, while the incorporation of additional independent variables led to moderate gains in the explanatory indices, it also substantially reduced parametric plausibility and reliability. The reduction in structural soundness may be attributed to the high degree of intercorrelation that characterized the development factors being considered. Table 4 illustrates this for bivariate and multivariate regression equations yielding Development Rate V as a power function of four development factors.

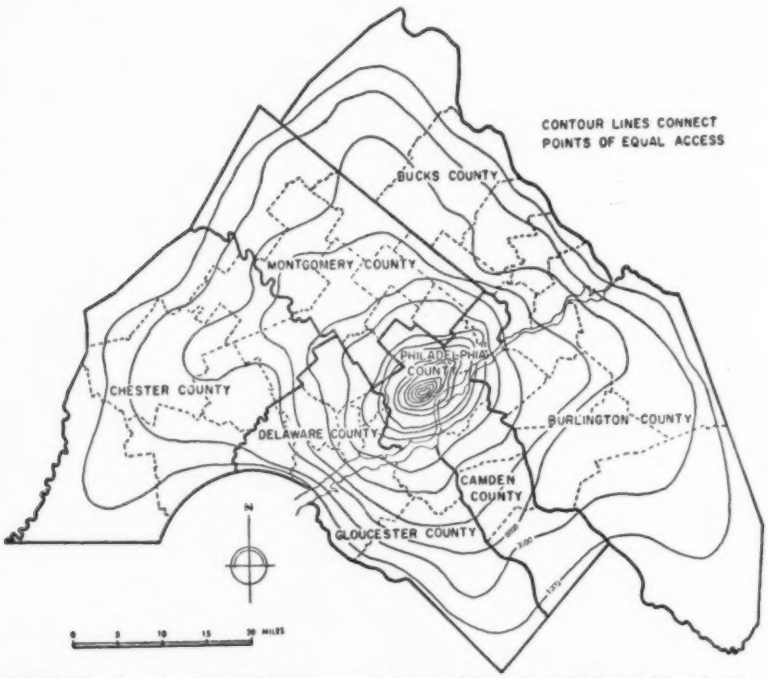

FIGURE 5: Levels of Total Employment Access (E), 1954–5; Philadelphia S.M.S.A.

Corrected correlation coefficients ($\bar{r}$) between pairs of the independent variables cited in Table 4 registered the following high values in Table 5.

TABLE 5

	C	*M*	*E*
B	−.8310	+.8900	+.9200
C		−.9170	−.8840
M			+.9780

Figures 4 and 5 illustrate the similarity of distribution of Residential Settlement and Employment Access in the Philadelphia Metropolitan Region.

Application of sets of less intercorrelated independent variables might lead to gains in explanatory ability with a much smaller reduction in structural soundness. However, the formulation of such variables is complicated by the fact that most metropolitan phenomena are distributed concentrically around the center, and are therefore strongly associated with one another.

A comparison of the efficiency of two regression equation sets incorporating, respectively, "potential" and "travel cost" indices as independent variables clearly established the superiority of the "potential" construct as a measure of subregional aggregate access. Four distance exponent values (0.5, 1.0, 1.5, 2.0) were alternatively applied with both constructs. Every one of the four "potential" indices attained substantially higher explanatory completeness than any of the "travel cost" indices. Table 6 illustrates this for two series of bivariate regression equations incorporating employment access as their independent variable.

TABLE 6: Levels of $\bar{r}^2$ for Regression Equations Yielding Development Rate V as a Linear Function of "Potential" and "Total Friction" Indices of Employment Access: 1950-56: N=44

Value of Distance Exponent (θ)	"Potential" $\bar{r}^2$	"Total Friction" $\bar{r}^2$
0.5	.3755	.0000
1.0	.5120	.0205
1.5	.5375	.1166
2.0	.4653	.1750

Note: The employment access indices used in this particular comparison are restricted to 1954 manufacturing jobs.

Four alternative values of the distance exponent θ were applied with equations incorporating residence and employment "potential" as their in-

TABLE 7: Levels of $\bar{r}^2$ for Bivariate Regression Equations Yielding Development Rate V as a Function of "Potential" Indices Incorporating Alternative Distance Exponents, 1950-56; N=44

	Linear	*Exponential*	*Power*
Residence "Potential" Index			
θ=0.5	.4062	.7755	.6557
θ=1.0	.5828	.7929	.7654
θ=1.5	.6821	.7319	.8183
θ=2.0	.7229	.5413	.8122
Employment "Potential" Index			
θ=0.5	.3768	.7477	.6077
θ=1.0	.5393	.7616	.7196
θ=1.5	.6088	.6803	.7814
θ=2.0	.6090	.5603	.7845

Note: The residence and employment "potential" indices express subregional aggregate access to, respectively, 1950 population and 1954/55 total jobs.

dependent variable. While the four alternative values (0.5, 1.0, 1.5 and 2.0) were selected essentially "a priori", their range is in line with values adopted for the "potential" indices of several previous models. Thus, the intra-national location models of Carrothers [5] and Stewart and Warntz [17] contain exponent values of 0.5, 1.0 and 2.0 in the former cases, and 1.0 in the latter. Again, Walter G. Hansen's [12] urban residential location model uses an exponent value of 2.20 with employment "potential", and a value of 2.35 with population "potential".

The relative efficiency of regression equations embodying the four alternative distance exponents was generally consistent for the several opportunity categories, but varied by equation form. With the linear and power function types, "potential" indices incorporating larger exponents (1.5 and 2.0) tended to perform better. With the exponential type, smaller exponents (0.5 or 1.0) tended to give higher levels of accuracy. (See Table 7)

4. SUMMARY

Metropolitan residential extension was viewed in terms of the residential development levels of metropolitan subregions, and a series of models yielding subregional development levels was described. The individual models incorporated alternative function types, development rates and development factors.

The models were empirically investigated by the regression method with data for the Philadelphia Metropolitan Region during the periods 1940–50 and 1950–56. The efficiency of the resulting regression equations was found to vary significantly by function type, development rate and development factor. Coefficient of determination levels ranged from zero to .8752.

REFERENCES

[1] Alonso, William. "A Theory of the Urban Land Market," *Papers and Proceedings of the Regional Science Association*, VI (1960).

[2] Blumenfeld, Hans. "Are Land Use Patterns Predictable" *Journal of the American Institute of Planners*, XXV (May, 1959), 61-66.

[3] Bogue, Donald J., and Harris, Dorothy L. *Comparative Population and Urban Research Via Multiple Regression and Covariance Analysis.* Published Jointly By the Scripps Foundation for Research in Population Problems at Miami University, and the Population Research and Training Center at the University of Chicago. Oxford and Chicago, 1954.

[4] Carrothers, Gerald A. P. "An Historical Review of the Gravity and Potential Concepts of Human Interaction," *Journal of the American Institute of Planners*, XXII (Spring, 1956), 94-102.

[5] _____. "Population Projection by Means of Income Potential Models," *Papers and Proceedings of the Regional Science Association*, IV (1958). Philadelphia. 121-152.

[6] Clark, Colin. "Urban Population Densities, "*Journal of the Royal Statistical Society*, Series A, CXIV, Part 4 (1951), 490-96.

[7] Duncan, Otis, Dudley. "The Measurement of Population Distribution," *Population Studies*, XI (July, 1957), 27-45.

[8] Dunn, Edgar S. "The Market Potential Concept and the Analysis of Location," *Papers and Proceedings of the Regional Science Association*, II (1956). Cambridge, Mass. 183-194.

[9] Ezekiel, Mordecai. *Methods of Correlation Analysis.* Second Edition. New York: John Wiley & Sons, 1941.

[10] Hagood, Margaret Jarman, and Price, Daniel O. *Statistices for Sociologists.* Revised

edition. New York: Henry Holt Co., 1952.
[11] Hamburg, John R., and Creighton, Roger L. "Predicting Chicago's Land Use Pattern," *Journal of the American Institute of Planners*, XXV (May, 1959), 67-72.
[12] Hansen, Walter G. "How Accessibility Shapes Land Use," *Journal of the American Institute of Planners*, XXV (May, 1959), 73-76.
[13] Herbert, John D., and Stevens, Benjamin H. "A Model for the Distribution of Residential Activity in Urban Areas," *Journal of Regional Science* (Fall, 1960), 21-36.
[14] Mills, Frederick C., *Statistical Methods Applied to Economics and Business*. Revised Edition. New York: Henry Hart & Co., 1938.
[15] Philadelphia City Planning Commission. *Land Use in Philadelphia 1944-1954*. Philadelphia, 1956.
[16] Schmitt, Robert C. "A Method of Projecting the Population of Census Tracts, *Journal of the American Institute of Planners*, XX (Spring, 1954), 102.
[17] Stewart, John Q., and Warntz, William. "Macrogeography and Social Science," *Geographical Review*, XLVIII (April, 1958) 167-184.
[18] Thomas, Edwin N. "Areal Association Between Population Growth and Selected Factors in the Chicago Urbanized Area," *Economic Geography*, XXXVI (April, 1960), 158-170.
[19] U. S. Bureau of the Budget. *Standard Metropolitan Statistical Areas*. 1961.
[20] U. S. Bureau of the Census. *1956 National Housing Inventory, Vol. I, Components of Change 1950 to 1956*, Part 1, United States and Regions, 1958; Part 9. Philadelphia Standard Metropolitan Area, 1958.
[21] U. S. Bureau of Labor Statistics *Structure of the Residential Building Industry in 1949*. Bulletin No. 1170, November, 1954.

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WAREHOUSING WITH TRANSHIPMENT UNDER SEASONAL DEMAND[1]

by Charles H. Kriebel

1. INTRODUCTION

The consideration of transhipment in linear programming transportation models has provided a more realistic analysis of warehouse activities for distribution planning. The primary advantage is that the treatment of warehouses as intermediate points—located between the origins (factories) and destinations (customers)—allows the provision for indirect customer shipments in analyzing distribution operations. Previous reports[2] on warehousing under seasonal demand for the most part have stressed dynamic programming techniques excluding direct consideration of the transhipment alternative as formulated by Orden [14]. The purpose of this paper is to indicate how transhipment of a seasonal product could be included in a warehouse planning model. In addition some modifications of the general formulation are discussed as suggestions for improving the model.

2. TRANSHIPMENT AND THE TRANSPORTATION PROBLEM

The standard "transportation problem"[3] in linear programming is the special ase which involves the selection of shipping routes for transferring known quantities of a single commodity from a number of specified origins to a number of specified destinations such that the total distribution cost is minimized. Under this class of problems only direct shipments are permitted in that each location acts as a shipper only or a receiver only. Mathematically, the problem is of the form

Minimize:

$$Z = \sum_{i=1}^{m} \sum_{j=m+1}^{m+n} c_{ij} x_{ij} \,. \tag{1.1}$$

(total cost of all shipments);

Subject to:

$$\sum_{i} x_{ij} = r_j \qquad \text{for } i = 1, \cdots, m \quad j = m+1, \cdots, m+n \tag{1.2}$$

1. The background research for this report was prepared in part while the author was affiliated with E. I. duPont de Nemours & Co. (Inc.), Wilmington, Delaware. The author wishes to thank E. P. Lotto of the ORCHEM Dept. for his support and interest during this period. For obvious reasons, particular specifications have been modified.

2. Bellman [1], Charnes and Cooper [3], Jewell [9], Levin, et.al. [12], Robacher [16], and Wester [17].

3. Dorfman, et. al., [6], Hitchcock [7], and Kantorovich [10] and [11].

(Total requirements at each destination must equal total shipped to it);

$$-\sum_{j} x_{ij} = k_i \tag{1.3}$$

(total shipped from each origin must equal[4] total capacity of the origin);

$$x_{ij} \geqq 0 \qquad \text{all } c_{ij} > 0 \tag{1.4}$$

(all shipments and unit costs are positive values);

where x_{ij} is the amount of the commodity shipped from origin i to location j;
c_{ij} is the transportation cost per unit from i to j, $i \neq j$;
r_j is the amount required at destination j; and
k_i is the amount possessed by origin i (capacity of factory i).

That is, solve the above for the x_{ij} values; given the values of the c_{ij}, r_j, and k_i.

In relaxing the direct shipment restriction to allow transhipment, the original conditions are extended so that "$\cdots$ shipments may go via any sequence of points rather than $\cdots$ (only via) $\cdots$ direct connections from one of the origins to one of the destinations."[5] That is, any shipping or receiving location may act as an intermediate point in an optimum (or minimum cost) solution.

Various approaches to the simple transhipment problem in programming or network format have been discussed.[6] However, for convenience it has been suggested[7] that Orden's original formulation provides an approach for solving the problem in a practical fashion (i.e., yields a program of feasible dimensions). Following the notation under the direct transportation problem, this treatment takes the form[8]

Minimize:

$$\bar{Z} = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{c}_{ij} x_{ij} \tag{1.5}$$

Subject to:

$$\sum_{i} x_{ij} = r_j + S \qquad \text{for } i, j = 1, \cdots, N \quad N = m + n \tag{1.6}$$

$$-\sum_{j} x_{ij} = -k_i - S \tag{1.7}$$

$$x_{ij} \geqq 0 \qquad \text{and } \bar{c}_{ij} > 0 \text{ for } i \neq j;\ \bar{c}_{ij} = 0 \text{ for } i = j; \tag{1.8}$$

(cost of transhipment from a point to itself is zero for all points)

where

4. More commonly this restriction might be stated as an inequality, i.e.,

$$\sum_{j} x_{ij} \leqq k_i \text{ or } -\sum_{j} x_{ij} \geqq -k_i$$

(total shipped from each origin cannot exceed the capacity of that origin).

5. Orden [14].

6. Bellman [1], Charnes and Cooper [3], Jewell [9], Levin, et. al., [12] Quandt [15], Robacher [16], and Wester [17].

7. Quandt [15].

8. *Ibid.*, 38-39.

$$S \geq \sum_{j=m+1}^{N} r_j , \qquad r_j = 0 \text{ for } j = 1, \cdots, m; \tag{1.9}$$

$$k_i = 0 \text{ for } i = m+1, \cdots, N .$$

(the positive constant S representing an artificial[9] inventory at each point). That is, solve the above *transportation* problem as before for the x_{ij} values; given the values of the $\tilde{c}_{ij}$, r_j, and k_i.

A brief comparison betwen the two analyses reveals that under the transhipment formulation the origins and destinations of the problem lose their identities as such, and both sets are considered only as locational designations in the system. More specifically, all points are treated as locations or cities that can receive, ship, and store the commodity.

The basic problem that this study is concerned with is the determination of the most efficient (i.e., least cost) means for warehousing and shipping a particular seasonal product *over a twelve month period.* The discussion that follows deals primarily with this decision paradigm in general terms for a specified distribution system. Variations and modifications of the model to include "more realistic" combinations are dealt with under the concluding comments.

In the interests of brevity an extended treatment of logistics in distribution planning is omitted in this paper,[10] and the discussion is confined to specific distribution problems, given the existence of a marketing system. Furthermore, to isolate the discussion it is assumed throughout that the elements of production (scheduling, balance, etc.) would be optimized for the system in general. While this assumption may be criticized as too unrealistic, it should be noted that many market planning decisions are made, in fact, with this tacit restriction. However, as will become apparent from the development, an extension of the model to include production planning for the system could be incorporated with little difficulty.

3. GENERAL SPECIFICATION OF THE PROBLEM

The problem may be stated as follows: If the total cost of distribution is represented by the transit and storage cost of the system, plan the product distribution for the entire year at minimum cost with "no lost sales."[11]

To provide a frame of reference for the above planning problem a hypothetical marketing system is introduced as follows: Consider a seasonal product inventory distributed through a nation-wide system of warehouses to different customers with different demand rates. For the present[12] it is assumed that the aggregate distribution of demand by customer locations is known in advance for each month throughout the year, and that sales transactions occur once per month on the same day each month. There is one fixed source of production for the system, the production process is continuous, and production is maintained

9. This artificial inventory or stockpile at each point is generated for computational purposes only and drops out of the final programming solution. Equations (1.6) and (1.7) are written as equalities assuming the necessary disposal activities have been satisfied.

10. Holt, et. al., [8], and Magee [13].

11. Where "no lost sales" is taken to mean no loss of sales during the year resulting from a deficiency in available supply (at the given level of monthly production)—See below.

12. See section 5.

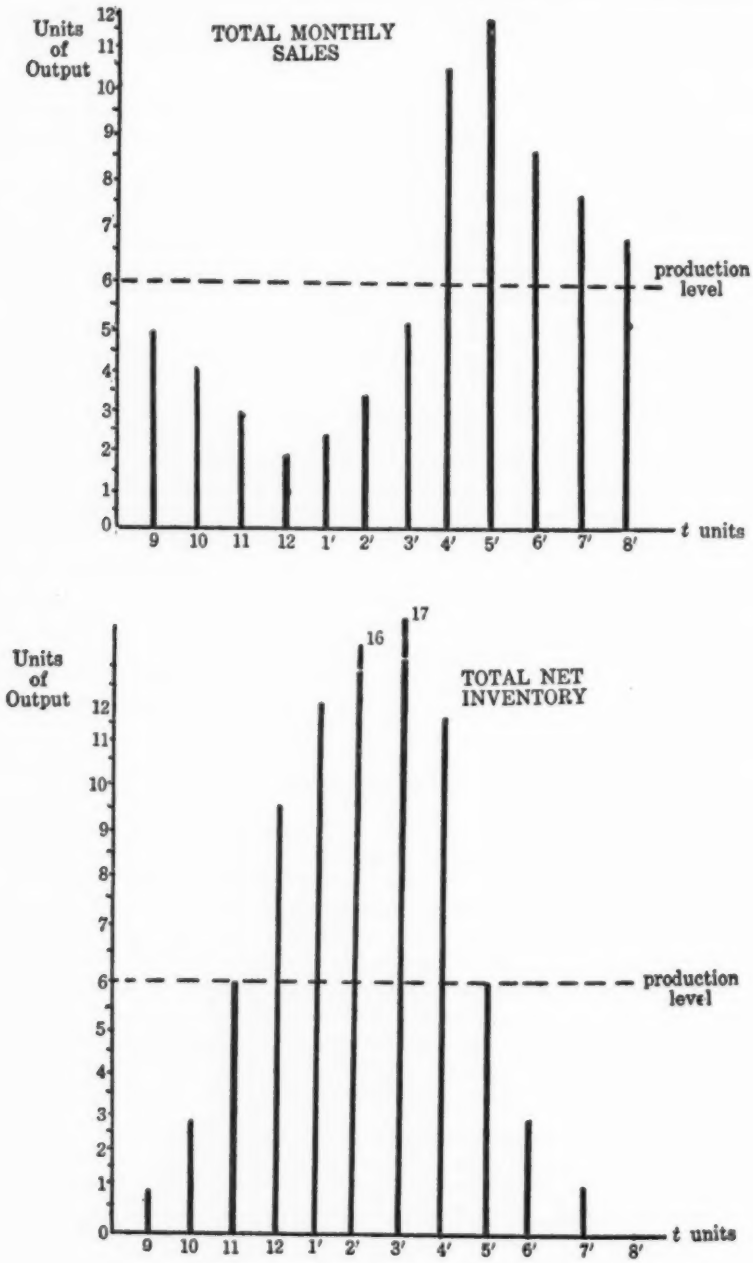

FIGURE 1. Aggregate Seasonal Pattern

at a constant level throughout the year. Total year's supply equals total year's demand for any year. Figure 1 indicates the scope of the problem in relative terms. Because of the high seasonal peak during which demand exceeds production, out-of-inventory sales occur from pre-season production that has been stored in the system's warehouses. Transportation and transhipment costs (per unit product) are known for all locations, as are warehouse (location) storage costs per unit product. It is assumed that all warehouse locations have "unlimited capacity" (e.g., all facilities are rented by the company and additional storage space can be made available for additional cost), however, the monthly storage costs per unit vary substantially between locations.

The alternatives of distribution for the system are represented in the network as: (1) a direct shipment from a plant to a customer (standard transportation problem), (2) a shipment from plant to a warehouse and from the warehouse to the customer ("static" transhipment); or (3) a delayed shipment or transhipment to the customer ("dynamical" transportation or transhipment).

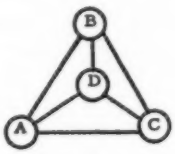

FIGURE 2. Alternatives of Distribution

Figure 2 illustrates these alternatives for any four points in the system, where any one (e.g., A) is taken to be a plant, any two (e.g., B and D) are taken as warehouses, and the remaining point (C) is taken as a customer location—on the assumption that the previous period's production has been available for system distribution. That is, at some starting moment in time the first period's output is "physically" located at the originating plant; thus only alternatives (1) and (2) are admissable (i.e., from A to C, or from A to D to C or A to B to C). However, once the system has been in operation for at least one time period all "three" alternatives can be considered (i.e., A to A-A to C, A to D-D to C, A to B-B to D to C, A to D to C-C to C, and so on). For this reason the artificial time designation of months (i.e., the abscissa) in Figure 1 begins during the "slack" season to allow for the generation of inventory stock in the system. After "time" has elapsed for the system as a whole, the original designation of the points "disappear," and all locations are treated as both potential supply and potential demand depots.

4. MODEL FORMULATION

To simplify this discussion we will first consider the case of a dynamical transportation model, and then proceed to a dynamical transhipment representation. Assuming at the outset that the costs of distribution are constant,[13] as is the usual case in the standard transportation problem, the problem alternatives

13. That is, all c_{ij} elements are specified constant cost functions and not a "function of the volume to be shipped." For considerations where this assumption is not appropriate see the literature on *non-linear* transportation problems.

for the first time period may be represented in the form of a matrix of costs for each shipping alternative. Consider the matrix $[c_{ij}]$ which has m rows and n columns (i.e., for origins $i = 1, \cdots, m$ and destinations $j = m + 1, \cdots, m + n$), where the element c_{ij} represents the transportation cost per unit from i to j—$[c_{ij}]$ constant throughout the total time period of consideration. A convenient dynamical expression of this shipping cost matrix $[c_{ij}]$ which includes the time dimension elements can be obtained by expanding the format into a much larger matrix. That is, the matrix $[c_{ij}]$ can be expanded row-wise over a time frame by considering first changing demand periods while the supply period is held constant—yielding a new Row 1. By a similar technique we can expand the columns; the resulting rectangular matrix would resemble the format of Table 3, below, where the original $[c_{ij}]$ would appear as sub-matrices along the main diagonal. For the standard dynamical transportation problem with m rows ($i = 1, \cdots, m$), n columns ($j = m + 1, \cdots, m + n$), and T time periods ($t = 1, 2, \cdots, T$) a shipment alternative matrix of ($Tm \times Tn$) dimensions is obtained which has ($T(m + n) - 1$) shipments as basic Min. solution elements.[14] If "back-ordering" is excluded as a distribution alternative the expanded square matrix reduces to a triangular matrix where all sub-matrices lying below the main diagonal contain infinite cost elements only, i.e., for all A, B such that $A > B$, $c_{ij}\ (A, B) = \infty$.[15] The notation $c_{ij}\ (A, B)$ is used to identify elements in the expanded matrix array; where A represents the supply period (row), B represents the demand period (column), and c_{ij} represents the shipping cost as before—A, B considered over the range $t = 1, \cdots, T$. For all sub-matrices such that $A = B$, i.e., the main diagonal (matrices) of the expanded array, $[c_{ij}(A, B)] = [c_{ij}]$ from the standard transportation problem. For all sub-matrices such that $B = A + 1$ in t, $[c_{ij}(A, B)] = [c_{ij}(A, B - 1)] + [s_{ij}]$; where $[s_{ij}]$ represents and $m \times n$ matrix whose elements s_{ij} indicate storage or holding costs per time period t at each location i as ($j = m + 1, \cdots, m + n$) over the range ($i = 1, \cdots, m$).[16] Finally, for all $k = (t - 1)$, ($t = 2, \cdots, T$), such that $B - A = 1$, $[c_{ij}(A, B)] = [c_{ij}(A_{k+1}, B_{k+2})] = [c_{ij}(A_k, B_{k+1})]$. Note that since the extended form of the problem is still of standard programming format any of the common computational procedures[17] can be used to determine a solution.

Broadening the above analysis to permit transhipment follows similar though not identical lines of develpment. As before, the new matrix $[\bar{c}_{ij}]$ is derived by extending the range for ($i, j = 1, \cdots, N$) where $N = (m + n)$, and for $A = B$, $\bar{c}_{ij} > 0$ if $i \neq j$, $\bar{c}_{ij} = 0$ if $i = j$. Now consider the assumption: $\bar{c}_{ij} = \bar{c}_{ji}$ for all $i \neq j$ in time periods $A = B$ (the shipping costs between any two particular points in the system are the same regardless of shipment direction if shipped during the same time period, for any i and j where $i \neq j$).[18] That

14. Note: Under the simple standard transportation problem with m rows and n columns the number of direct routes required in a basic Min. cost solution for non-degenerate cases is $(m + n - 1)$.

15. That is, this represents an easy way to exclude the "back-ordering" alternative from the problem.

16. Note: Storage costs are a constant increasing linear function of time, i.e., by constant discrete increments; whereas transportation costs are constant per time period, i.e., the same during each sub-period over the total time range.

17. Charnes and Cooper [2], and Dantzig [4] and [5].

is, the new matrix $[\tilde{c}_{ij}]$ is symmetric, or $[\tilde{c}_{ij}] = [\tilde{c}_{ji}]^{\mathrm{T}} = [\tilde{c}_{ij}]$ (where the notation $[\tilde{c}_{ij}]^{\mathrm{T}}$ represents the transpose of $[\tilde{c}_{ij}]$). The significance of this symmetry assumption in simplifying computations becomes apparent when one considers the number of combinatorial alternatives present in the transhipment specification over time. Under the direct transportation problem the movement of commodities is uni-directional, i.e., from i to j only, and the identification of system points as either origins or destinations (customer locations) is critical to the analysis. As a result commodity storage facilities are taken to exist only at the origins in the system, and for the delayed shipments of the expanded matrix holding costs are calculated at i designations only. That is, all column elements for any particular row i of $[s_{ij}]$ are identical, e.g., for $j = m + 1, \cdots, m + n$: $\tilde{c}_{1j}(A, B + 1) - \tilde{c}_{1j}(A, B) = s_{1j} = s_1$, etc. Thus, to indicate the alternative of satisfying demand in period 6 with production (supply) from period 2, the cost designation $\tilde{c}_{ij}(2, 6)$ represents the storage costs for four (t) periods at location i plus the direct transportation costs from i to j. However, once transhipment is introduced into the model, the above analysis no longer applies, since the system points are no longer identified as "origins" or "destinations". A delayed shipment from i to j for one period, therefore, can be stored at either i *or* j for one period and shipped accordingly, i.e., either (1) stored at i for one period and then shipped to j, or (2) shipped from i to j and then stored at j for one period. Note that this alternative was not present under the extended form of the direct transportation problem—and was excluded under the simple transportation problem since customer j's second period demand is taken as nonexistent during period one. Without assuming a symmetric $[\tilde{c}_{ij}]$ the expansion of the matrix to include time dimension elements would require a "double" extension of $[\tilde{c}_{ij}]$ for each dimension, i and j (i.e., an expansion row-wise *and* column-wise for changing demand periods with the supply period held constant, and similarly for changing supply periods with the demand period constant). At a minimum this would yield a matrix with twice as many rows as the previous array, and would necessitate a different interpretation for the respective elements of the new expanded array. Admitting the symmetry assumption permits the analysis to follow essentially as under the direct transportation problem, i.e., if $\tilde{c}_{ij}(A, B) = \tilde{c}_{ji}(A, B)$ for $A = B$, then for $i < j$, $\tilde{c}_{ij}(A, B + 1) = \tilde{c}_{ij}(A, B) + s_{(i<j)}$, and for $i > j$, $\tilde{c}_{ij}(A, B + 1) = \tilde{c}_{ij}(A, B) + s_{(i>j)}$, where $i > j$ represents sub-matrix elements above the expanded array's main diagonal, etc.

As noted under the introductory remarks, the simple transhipment problem is solved as a standard transportation problem via any of the normal calculation routines. The symmetry assumption allows the expanded transhipment format to be solved as an expanded transportation problem, and will also contain

18. This assumption may be regarded by some as too limiting for many instances of analysis. This would be particularly true if the mode of transportation were railroad carriers where freight rates are less per unit for east-to-west traffic than for west-to-east traffic, and similarly for north-south shipments. However, if commercial trucking carriers are considered this discrepancy tends to be reduced, and where the mode of transport is owned and operated by the producing firm the differential in cost disappear altogether for practical purposes. Generally, even for extremes of direction (and distance) by rail, the i-to-j vs. j-to-i differential rarely exceeds a constant multiple $\doteqdot 2$.

$(T(m+n)-1)$ shipments as basic Min. solution elements.[19] We may summarize this discussion by specifying the problem as of the form
Minimize:[20]

$$\tilde{Z} = \sum_{t,t'=1}^{T} \sum_{i,j=1}^{N} \tilde{c}_{ti,t'j} x_{ti,t'j} \tag{3.1}$$

subject to:[21]

$$\sum_{ti} x_{ti,t'j} = r_{t'j} + S \qquad \begin{matrix} i,j = 1,\cdots,N \\ N = m+n \\ t,t' = 1,\cdots,T \end{matrix} \tag{3.2}$$

$$-\sum_{t'j} x_{ti,t'j} = -K_{ti} - S \tag{3.3}$$

$$x_{ti,t'j} \geqq 0 \tag{3.4}$$

$$\tilde{c}_{ti,t'j} = \tilde{c}_{t'j,ti} > 0 \text{ for } i \neq j \text{ and } t = t' \qquad (t,t' = 1,\cdots,T) \tag{3.5}$$
$$\tilde{c}_{ti,t'j} = 0 \text{ for } i = j,\ t = t' .$$

To exclude "back ordering" add the restriction

$$\text{all } \tilde{c}_{ti,t'j} = \infty \text{ for } ti > t'j \qquad \begin{matrix} t,t' = 1,\cdots,T \\ i,j = 1,\cdots,N \end{matrix} \tag{3.6}$$

and

$$\tilde{c}_{ti,t'j} = \tilde{c}_{ti,(t'-1)j} + s_{ti,j} \text{ for } ti < t'j .$$

5. A HYPOTHETICAL EXAMPLE

To illustrate the previous discussion a small problem will be developed. Consider a distribution system that entails seven geographic locations—e.g., one plant, two warehouses, and four customer areas as a direct transportation problem. The direct transportation (shaded area) and transhipment cost matrix, $[\tilde{c}_{ij}]$, is given in Table 1 (A). For convenience the operation of this system is considered over four time periods only (e.g., months or calendar quarters). Storage costs for each location per time period (t) are given in Table 1 (B). Alternatively, the static alternatives of the problem may be represented pictorially by a network diagram, as indicated by Figure 3.

Analyzing the problem as a static transportation or transhipment problem (i.e., for one time period only) yields the solution indicated by the matrices in Table 2. (Note: the transhipment problem for the static case is given by Table 1 (A).) Under the single static alternative the inclusion of transhipment reduces total costs by 7.70.

19. Orden [14], p. 283: The transhipment problem, dealing with N points where $N = m+n$, makes use of N origins and N destinations, and leads initially to a solution involving $(2N-1)$ paths. Of these, however, N are X_{ii} terms which are reduced to zero. Thus the final solution makes use of $(N-1) = (m+n-1)$ paths which is the same as the standard transportation problem. (See the text at the beginning of this section).

20. Note: From the preceding discussion the row index $A = t$ in the expanded matrix, and the column index $B = t'$; $(t,t' = 1,\cdots,T)$ of the model.

21. Where S is computed as before (1.9) modified only by extending the range for i and j by T.

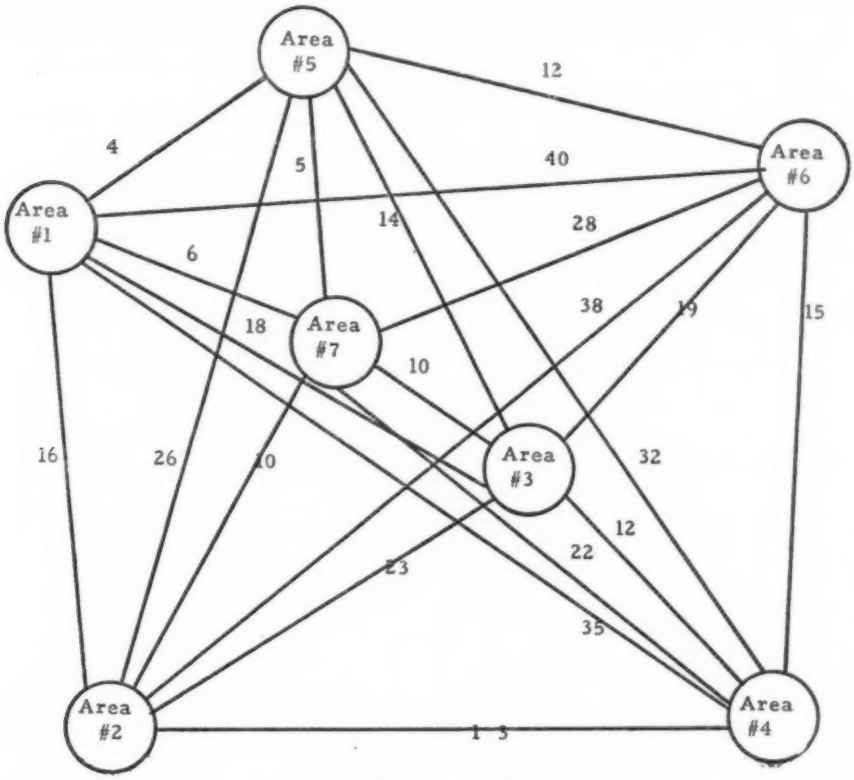

FIGURE 3. Network Diagram

TABLE 1: Allocation Cost Matrices

		1 (A)								1 (B)			
Origins:	K_1	0	16	18	35	4	40	6		K_1	2	...	2
	K_2	16	0	23	13	26	38	10		K_2	8	...	8
	K_3	18	23	0	12	14	19	10		K_3	12	...	12
	K_4	35	13	12	0	32	15	22		K_4	9	...	9
							c_{ij}					a_{ij}	
	K_5	4	26	14	32	0	12	5		K_5	9	...	9
	K_6	40	38	19	15	12	0	28		K_6	5	...	5
	K_7	6	10	10	22	5	28	0		K_7	13	...	13
		r_1	r_2	r_3	r_4	r_5	r_6	r_7			r_1	...	r_7

Destinations

Broadening the analysis to include seasonal demand requirements over time yields the problem specification in Table 3. Total production for each period

was taken to be 360 units; and total sales for each of the four periods were 240, 120, 660, and 420, respectively. Once the problem has been specified in the form of Table 3, the solution can be determined as under the static case, via any convenient computational routine.

In the interests of brevity a detailed discussion of the illustrative example

TABLE 2: Static Problem

2 (A)
Direct Shipment Problem

$c_{ij}\rightarrow$	4	26	14	32	K_5
	40	38	19	15	K_6
	6	10	10	22	K_7
	r_1	r_2	r_3	r_4	

K_i: 1 : m (m) amounts to be shipped

r_j: $m+1 \cdots\cdots m-n$ (n) amounts received

2 (B)
Min. Cost Solution to Direct Problem

	20	70	60	30	180 $\leftarrow K_i$
$x_{ij}\rightarrow$	0	0	0	90	90
	0	90	0	0	90
$r_j\rightarrow$	20	160	60	120	

(Total Cost Direct Transportation Problem=59.50)

2 (C)
Min. Cost Solution to Transhipment Problem

	700	0	0	0	0	0	0	700 $\leftarrow K'_i=1$
	0	700	0	0	0	0	0	700 $i=2$
	0	0	700	0	0	0	0	700
$x'_{ij}\rightarrow$	0	0	0	700	0	0	0	700
	20	0	60	30	700	0	70	810
	0	0	0	90	0	700	0	790
	0	160	0	0	0	0	630	790 $m+n\ (=N)$
$r'_i\rightarrow$	720	860	867	820	700	700	700	

$i=1 \cdots\cdots m+n\ (=N)$

2 (D)
Final Solution to Transhipment Problem

								K''_i	
	0	0	0	0	0	0	0	0	0 $\leftarrow K^0_i$
	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
$x''_{ij}\rightarrow$	0	0	0	0	0	0	0	0	0
	20	0	60	30	0	0	70	180	180
	0	0	0	90	0	0	0	90	90
	0	160	0	0	0	0	0	160	90
$r''_i\rightarrow$	20	160	60	120	0	0	70		
$r^0_i\rightarrow$	20	160	60	120	0	0	0		

(Total Cost (Static) Transhipment Problem=51.80)

TABLE 3: Warehousing Model Prototype. (Allocation Cost Matrix)

SUPPLY PERIOD ($t=A$)	($t'=B$) 1							2 DEMAND PERIOD 3							4							k_t
	0	16	18	35	4	40	6	2	18	20	37	6	42	8	4	20	22	39	8	44	10	6	22	24	41	10	46	12	0
	16	0	23	13	26	38	10	24	8	31	21	34	46	18	32	16	39	29	42	54	26	40	24	47	37	50	62	34	0
	18	23	0	12	14	19	10	30	35	12	24	26	31	22	42	47	24	36	38	43	34	54	59	36	48	50	55	46	0
1	35	13	12	0	32	15	22	44	22	21	9	41	24	31	53	31	30	18	50	33	40	62	40	39	27	59	42	49	0
	4	26	14	32	0	12	5	13	35	23	41	9	21	14	22	44	32	50	18	30	23	31	53	41	59	27	39	32	180
	40	38	19	15	12	0	28	45	43	24	20	17	5	33	50	48	29	25	22	10	38	55	53	34	30	27	15	43	90
	6	10	10	22	5	28	0	19	23	23	35	18	41	13	32	36	36	48	31	54	26	45	49	49	61	44	67	39	90
								0	16	18	35	4	40	6	2	18	20	37	6	42	8	4	20	22	39	8	44	10	0
								16	0	23	13	26	38	10	24	8	31	21	34	46	18	32	16	39	29	42	54	26	0
								18	23	0	12	14	19	10	30	35	12	24	26	31	22	42	47	24	36	38	43	34	0
2								35	13	12	0	32	15	22	44	22	21	9	41	24	31	53	31	30	18	50	33	40	0
								4	26	14	32	0	12	5	13	35	23	41	9	21	14	22	44	32	50	18	30	23	180
								40	38	19	15	12	0	28	45	43	24	20	17	5	33	50	48	29	25	22	10	38	90
								6	10	10	22	5	28	0	19	23	23	35	18	41	13	32	36	36	48	31	54	26	90
															0	16	18	35	4	40	6	2	18	20	37	6	42	8	0
															16	0	23	13	26	38	10	24	8	31	21	34	46	18	0
															18	23	0	12	14	19	10	30	35	12	24	26	31	22	0
3															35	13	12	0	32	15	22	44	22	21	9	41	24	31	0
															4	26	14	32	0	12	5	13	35	23	41	9	21	14	180
															40	38	19	15	12	0	28	45	43	24	20	17	5	33	90
															6	10	10	22	5	28	0	19	23	23	35	18	41	13	90
																						0	16	18	35	4	40	6	0
																						16	0	23	13	26	38	10	0
																						18	23	0	12	14	19	10	0
4																						35	13	12	0	32	15	22	0
																						4	26	14	32	0	12	5	180
																						40	38	19	15	12	0	28	90
																						6	10	10	22	5	28	0	90
	8	64	24	48	40	32	24	4	32	12	24	20	16	12	22	176	66	132	110	88	66	14	112	42	84	70	56	42	

PRODUCTION

REQUIREMENTS

will not be undertaken beyond this specification, since the conclusions should appear obvious. However, it may be appropriate to comment on some of the features that arise in the example as a consequence of the time dimension. For example, it will be noted that $c_{55}(1, 3) = 18 > c_{51}(3, 3) = 4 + c_{15}(1, 3) = 8$. However, $c_{55}(1, 2) = 9 < c_{51}(2, 2) = 4 + c_{15}(1, 2) = 6$. Thus, taking $i = 5$ as the originating plant, it is less expensive to tranship surplus inventory from the plant during period 1, to location $i = 1$ for storage, and then to reship back to the plant to satisfy the seasonal peak demand of period 3, at the plant, than to hold this inventory at the plant for two periods. Similarly, while during the first time period (see static example solution) it was cheaper to tranship from 5-to-7-to-2 than to ship direct from 5-to-2, after one time period has elapsed, however, it is cheaper to tranship from 5-to-1, (store at 1), and then ship either from 1-to-7-to-2 *or* from 1-to-2.[22] Similar comments can be made with respect to the dynamical transhipment distribution that is introduced in proceeding from period 3 to period 4.

Before concluding the discussion it should be noted that every linear programming problem has a *dual* problem associated with its direct specification. While there is no intention to elaborate on this point,[23] a liberal interpretation of the dual elements in this formulation would be that they represent the "economic rent" accruing to a particular location in the system. For purposes of analysis these elements might be of interest to a marketing planner in valuing capacities, etc.—particularly for such considerations as the future expansion of existing facilities.

6. FURTHER COMMENTS

Although the problem discussed in this paper was subjected to some important specification restrictions, nearly all[24] of these are not critical to the analysis and were introduced primarily to simplify the presentation.

The assumption concerning a constant level of production over time could easily be withdrawn to incorporate varying output levels directly into the model. An extension of this form would require a minor modification of the K_i values is the expanded matrix to allow for variations, in output in the producing facilities. To determine an optimal production schedule for the system as a whole, production supply limitation, optimal lot size specifications, and the costs of changing production levels would have to be considered. Any or all of these factors could be added to the model with little difficulty.

Recognition of "probabilistic" demand requirements could also be introduced, but again at the cost of additional detail. Under this treatment each of the corresponding r_j values over time would be interpreted with respect to probability density functions rather than actual location requirements (as was the case when the demand was known in advance). To illustrate briefly, if we exclude consideration of shortage costs—i.e., no provision for the cost of lost sales—the respective r_j values could be interpreted as the central tendency index, such as the arithmetic mean, of each location's demand density. The probabilities, etc., would have to be known in advance and the overall problem would then be to minimize: the total *expected* cost of distribution. If shortage costs were to be included the specification would become more complicated.[25] Similar comments could be made for the introduction of multi-product lines, and non-linear distribution and/or storage costs.

The preceding analysis indicates that "dynamical" transhipment in a

22. That is, referring to Table 3:

$$c_{32}(1,1) > c_{31}(1,1) + c_{12}(1,1) > c_{37}(1,1) + c_{72}(1,1)\,,$$

however,

$$c_{32}(1,2) > c_{23}(1,2) > c_{37}(2,2) + c_{72}(1,2) > c_{31}(2,2) + c_{17}(1,2) + c_{72}(2,2) \text{ and}$$
$$c_{31}(2,2) + c_{17}(1,2) + c_{72}(2,2) = c_{31}(2,2) + c_{12}(1,2)\,.$$

23. See Dorfman, Samuelson, and Solow [6].

24. The symmetry assumption of the initial $\bar{c}_{ij}$ may be considered the exception for this analysis.

25. For example, the analyst would be dealing with a function of the ratio of distribution and shortage costs which would yield a correction factor—in probability terms—to be taken with some dispersion measure, e.g., the standard error. This value would then be applied to the central tendency index to determine an "optimum" expected requirement at each location. That is, if

$$\int_0^{I_j} f_j(x)dx = \frac{c_2}{c_1 + c_2} = \alpha_j\,, \quad \text{then } \bar{X}_j + a_j(\sigma_{2_j}) = I_j = r_j\,;$$

given: $f_j(x;\, \mu_j, \sigma^2{}_j)$, for $j = 1, \cdots, N$.

warehouse planning model provides for the consideration of additional distribution alternatives not available under the expanded direct transportation problem over time. The minimum cost solution in this case, as in the static example, is less than or equal to the total direct transportation cost solution, and requires no more shipment paths than would be needed if only direct shipment were permitted. In addition, the procedure is not restricted to warehousing or distribution problems, and may be applied to any combinatorial problem which is specified in programming format.

REFERENCES

[1] R. Bellman, "On the Theory of Dynamic Programming-A Warehouse Problem," *Management Science*, 2-3: 272-275 (April, 1956).

[2] A. Charnes, and W. W. Cooper, "The Stepping Stone Method of Explaining Linear Programming Calculations in Transportation Problems," *Management Science*, 1-1.

[3] A. Charnes, and W. W. Cooper, "Generalizations on the Warehousing Model," *O. N. R. Research Memorandum*, 34 (Carnegie Institute of Technology, 1955).

[4] G. B. Dantzig, in T. C. Koopmans (ed.): *Activity Analysis of Production and Allocation*, Ch. XXIII, (J. Wiley and Co., 1951).

[5] G. B. Dantzig, and P. Wolfe, "Decomposition Principle for Linear Programs," *Operations Research*, 8-1: 101-111 (Jan.-Feb., 1960).

[6] R. Dorfman, P. Samuelson, and R. Solow, *Linear Programming and Economic Analysis*, Ch. 5: 106-129 (McGraw-Hill, 1958).

[7] F. L. Hitchcock, "The Distribution of a Product from Several Sources to Numerous Localities," *Journal of Mathematics and Physics*, 20: 224-230 (1941).

[8] C. C. Holt, F. Modigliani, J. F. Muth, and H. A. Simon, *Planning Production, Inventories and Work Force*, (particularly Ch. 17) (Prentice Hall, 1960).

[9] W. S. Jewell, "Warehousing and Distribution of a Seasonal Product," *Naval Research Logistics Quarterly*, 4-1: 29-34 (March, 1957).

[10] L. V. Kantorovich, "On the Translocation of Masses," (1942), *Management Science*, 5-1 (October, 1958).

[11] L. V. Kantorovich, and M. K. Gavurin, "The Application of Mathematical Methods to Freight Flow Analysis," *Problems of Raising the Efficiency of Transport Performance* (Akademiia Nauk, U. S. S. R., 1949).

[12] B. Levin, B. Bender, and A. Goldman, "Optimization of Distribution Networks," *National Bureau of Standards Report*: 6390 (July, 1960).

[13] J. F. Magee, "The Logistics of Distribution," *Harvard Business, Review* 38-4: 89-101 (July-August, 1960).

[14] A. Orden, "The Transhipment Problem" *Management Science*, 2-3: 277-285, (April, 1956).

[15] R. E. Quandt, "Models of Transportation and Optimal Network Construction," *Journal of Regional Science*, 2-1: 27-45 (Spring, 1960).

[16] J. T. Robacker, "Concerning Multi-Commodity Networks, Notes on Linear Programming: Part XXXVII," *RAND Research Memorandum* (September, 1956).

[17] L. Wester, "Location-Allocation Models with Time Dependent Requirements at Destinations," (Armour Research Foundation of Illinois Institute of Technology) *Paper Presented at Annual Meeting of O. R. S. A.* (May, 1959)

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MEASUREMENTS OF MANUFACTURING: COEFFICIENTS OF CORRELATION†

by John W. Alexander and James B. Lindberg*

1. INTRODUCTION

For years scholars interested in analyzing spatial variation in manufacturing have speculated as to the best method for measuring the location of manufacturing. Several different methods have been tried. Sten de Geer, one of the first to construct maps showing the location of manufacturing, based his maps on *number of wage earners* [1]. Helen Strong chose the criterion *horse power of prime movers* [2]. Alfred Wright [3] and Victor Roterus [4] preferred *value added*. Gunnar Alexandersson [5] prepared what is probably the most detailed series of maps ever constructed of United States manufacturing, and for his measurement he selected *number of employees*. Each of the foregoing scholars based his analysis upon a single absolute measurement.

Other scholars have experimented with a combination of two or more criteria, especially by means of ratios whereby one absolute value is compared to another. Harold McCarty [6], John Thompson [7] and Wilbur Zelinsky [8] have experimented with such systems. An effort to define several different methods for measuring manufacturing has also been made by Alexander [9].

Frequently, when papers dealing with location of manufacturing have been presented at meetings of professional scholars (whether at seminars of graduate students or at national meetings of scientific associations), questions have been raised about the author's selection of criteria by which he measured the amount of manufacturing in the places studied. If he adopted *number of employees*, somebody was likely to propose that *value added* would have been a more satisfactory measurement.

And so, for years spatial analysts have speculated as to what criterion is best for measuring the location and the spatial variation in manufacturing.

2. PURPOSE

The purpose of the research reported herein is to investigate this general question: how much difference is there between criteria (for measuring manufac-

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turing) as they differ from place to place? Different scholars could propose different approaches, but the one chosen for this particular study was formulated as follows: what are the coefficients of correlation between the several possible pairs of manufacturing criteria as they vary spatially?

The study was launched with a certain assumption in mind: if a map based on *value added* shows the same locational pattern as a map based on *number of employees*, then these two criteria are equally useful for showing the location of manufacturing. Consequently the argument as to which of the two is the "better" criterion — for showing locational pattern—could be settled because both criteria give the same result.

3. PROCEDURE

The case study area chosen for this investigation was the United States. The study period was the year 1954, the date of the latest available *Census of Manufactures* [10]. The 1954 *Census of Manufactures* quantifies data on manufacturing under thirteen different headings: number of establishments, number of employees, value of payroll, number of production workers, value of wages, number of production man-hours worked, value added in the manufacturing process, capital expenditures in the year 1954, number of employees in the year 1947, value added in 1947, and three variables showing the number of firms by size class in 1954: those with less than 20 employees, those with from 20 to 99, and those with more than 100 employees.

With these data, it would be possible to construct thirteen maps of the United States showing the amount of manufacturing in each country — each map based on one of the thirteen measurements. *If* it should then develop that all thirteen maps had the same pattern then it would seem reasonable to conclude that (for purposes of showing spatial variation) it would be immaterial which of the thirteen absolute measurements were used.

But the construction of thirteen maps is a rather crude way of approaching this problem, for even if the maps were drawn it would be impossible to compare them except by visual inspection. This is not to depreciate the value of visual inspection for analysis of correlation between two or more areal patterns. However, it is impossible to come up with a precise measurement of variation by means of visual inspection alone.

Therefore, the decision was made to run a correlation analysis of the thirteen criteria and to compute coefficients of correlation for each pair of criteria as they varied from place to place within the United States.

The next decision involved choice of areal units: should manufacturing be measured in terms of its location by states, by state economic areas, by metropolitan areas, or by counties. The decision was made arbitrarily to base the study on county units, except where two or more counties constituted a single metropolitan area and were grouped together. On this basis, the study began with 2957 areal units, of which 2791 were individual counties and 166 were standard metropolitan areas.

Several counties caused trouble in statistical computation of coefficients of correlation. The first difficulty involved 45 counties which had no manufacturing whatsoever. Obviously it is impossible to come up with a meaningful measure of correlation between variables in counties where the variables are

non-existent. It was decided to omit these from the study.

A second more serious difficulty involved 766 counties for which the Census gave information for some but not all of the thirteen criteria. These were also deleted from the analysis. It is not known what effect the omission of these 766 counties had on the result obtained, but it is believed not to have been great. In most instances data were withheld because of the "disclosure rule" adopted by the Census. The remaining 1980 counties together with the 166 metropolitan areas gave an overall total of 2146 areal units upon which this research was finally based. This base number constitutes a 73% sample of the 2957 possible areal units.

Data for the 2146 areal units were put on punch cards which then were processed on an IBM 650. The Pearsonian coefficients of correlation were computed for each pair of variables over all 2146 areal units.

4. RESULTS

Nation as a whole: Table 1 shows the correlations of the thirteen criteria for the nation. In this table, each criterion is identified by both a number and initial letters.

TABLE 1: Measurements of Manufacturing, United States 1954. Coefficients of Correlation for 2146 Areal Units (Metropolitan Areas and Counties).

Criteria	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T····· (number of establishments)	·94	·92	·94	·93	·91	·92	·40	·92	·93	·99	·99	·93
2. E····· (number of employees)		·99	·99	·99	·99	·99	·62	·99	·99	·93	·93	·99
3. VEP····· (value of employee payroll)			·99	·99	·99	·99	·66	·98	·98	·91	·92	·98
4. PW····· (number of production workers)				·99	·99	·99	·62	·99	·99	·93	·93	·99
5. PMH····· (number of production man-hours)					·99	·99	·63	·99	·99	·92	·93	·99
6. VW····· (value of wages paid production workers)						·99	·68	·98	·98	·90	·91	·98
7. VA····· (value added)							·65	·99	·99	·91	·92	·98
8. CE····· (1954 capital expenditures)								·61	·60	·38	·59	·46
9. 47E····· (number of employees 1947)									·99	·91	·92	·98
10. 47VA····· (value added 1947)										·92	·93	·98
11. T-1····· (number of establishments 1954 with 1-19 employees)											·99	·92
12. T-20····· (number of establishments 1954 with 20-99 employees)												·93
13. T-100····· (number of establishments 1954 with 100 or more employees)												

With more than 2000 observations for the nation as a whole and with at least 53 observations in the smallest of the Census regions, tests for significance indicate that all the coefficients computed in this study are meaningful. That is, all the coefficients are sufficiently different from zero to be judged statistically significant at the ·01 level of probability.

Table 1 reveals that most of the coefficients exceed ·90. In fact, there is no place on the table where coefficients fall below ·90 except for criterion number 8, *capital expenditures in 1954*. This criterion deviates so sharply from the other twelve criteria that nowhere does its coefficient reach ·90.

The high correlations throughout most of this matrix make one wonder if, for purposes of mapping manufacturing for the nation as a whole, it is not immaterial which criterion is used — with the exception of criterion number 8, *capital expendituress in 1954*. Such low coefficients for criterion number 8 suggest that a separate map showing capital expenditures would be in order.

Individual Census Regions: The next question was whether such high correlations prevailed in each of several different sections of the country. Accordingly, a similar set of correlations was run for each of the nine census regions. Tables 2 through 10 contain the findings. (It is assumed that the reader is familiar with the extent of each Census Region.) Each table also indicates the number of areal units (broken down by number of metropolitan areas and by number of counties, exclusive of metropolitan areas) whose measurements were used in computing the coefficients.

Every region (except the *South Atlantic*) has a matrix which resembles that for the nation as a whole; that is, high coefficients tend to prevail for every pair of variables except in some cases when criterion 8 (capital expenditures) is involved. But in Table 6 (*South Atlantic states*) high correlations appear for only criteria 2, 3, 4, 5, 6, and 7. This matrix has several low coefficients for criteria 8, 9, 10, 11, 12 and 13 indicating considerable locational change of manufacturing in this region between 1947 and 1954.

TABLE 2: New England
Coefficients of Correlation
(53 areal units: 12 metropolitan areas, 41 counties and towns)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·97	·96	·97	·97	·95	·96	·90	·97	·98	·99	·99	·98
2. E		·98	·99	·99	·99	·99	·95	·99	·99	·97	·97	·98
3. VEP			·99	·99	·99	·99	·96	·99	·99	·95	·96	·97
4. PW				·99	·99	·99	·95	·99	·99	·96	·97	·99
5. PMH					·99	·99	·95	·99	·99	·96	·96	·98
6. VW						·99	·96	·99	·99	·95	·95	·97
7. VA							·95	·99	·99	·96	·96	·98
8. CE								·94	·94	·90	·89	·92
9. 47E									·99	·97	·98	·99
10. 47VA										·97	·98	·99
11. T-1											·99	·98
12. T-20												·99
13. T-100												

TABLE 3: Middle Atlantic
Coefficients of Correlation
(110 areal units: 21 metropolitan areas, 89 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·97	·97	·97	·97	·96	·97	·86	·97	·98	·99	·99	·97
2. E		·99	·99	·99	·99	·99	·95	·99	·99	·97	·97	·99
3. VEP			·99	·99	·99	·99	·95	·99	·99	·96	·97	·99
4. PW				·99	·99	·99	·94	·99	·99	·97	·97	·99
5. PMH					·99	·99	·95	·99	·99	·97	·97	·99
6. VW						·99	·96	·99	·99	·96	·97	·99
7. VA							·95	·99	·99	·97	·97	·99
8. CE								·95	·94	·85	·86	·95
9. 47E									·99	·96	·97	·99
10. 47VA										·97	·98	·99
11. T-1											·99	·96
12. T-20												·97
13. T-100												

TABLE 4: East North Central
Coefficients of Correlation
(375 units: 37 metropolitan areas, 338 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·99	·98	·98	·98	·97	·98	·89	·99	·99	·99	·99	·99
2. E		·99	·99	·99	·99	·99	·93	·99	·99	·99	·98	·98
3. VEP			·99	·99	·99	·99	·95	·99	·99	·98	·98	·97
4. PW				·99	·99	·99	·94	·99	·99	·98	·98	·97
5. PMH					·99	·99	·94	·99	·99	·98	·98	·97
6. VW						·99	·96	·99	·99	·98	·97	·96
7. VA							·94	·99	·99	·99	·98	·97
8. CE								·93	·92	·90	·89	·86
9. 47E									·99	·99	·98	·98
10. 47VA										·99	·99	·98
11. T-1											·99	·99
12. T-20												·99
13. T-100												

TABLE 5: West North Central
Coefficients of Correlation
(365 units: 16 metropolitan areas, 349 counties)

		2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1.	T	·98	·99	·98	·98	·98	·99	·96	·98	·99	·99	·99	·98
2.	E		·99	·99	·99	·99	·99	·97	·98	·99	·98	·98	·99
3.	VEP			·99	·99	·99	·99	·96	·98	·98	·98	·99	·98
4.	PW				·99	·99	·99	·97	·99	·99	·97	·97	·99
5.	PMH					·99	·99	·97	·98	·98	·97	·98	·99
6.	VW						·99	·97	·98	·98	·98	·98	·98
7.	VA							·97	·98	·99	·98	·99	·99
8.	CE								·98	·98	·95	·95	·98
9.	47E									·99	·97	·98	·99
10.	47VA										·98	·99	·99
11.	T-1											·98	·97
12.	T-20												·98
13.	T-100												

TABLE 6: South Atlantic
Coefficients of Correlation
(452 units: 27 metropolitan areas, 425 counties)

		2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1.	T	·87	·87	·86	·86	·86	·85	·72	·83	·82	·98	·98	·89
2.	E		·99	·99	·99	·99	·98	·83	·98	·98	·81	·92	·98
3.	VEP			·99	·98	·99	·99	·84	·98	·98	·81	·94	·95
4.	PW				·99	·99	·98	·83	·99	·98	·79	·89	·98
5.	PMH					·98	·97	·82	·98	·97	·80	·92	·98
6.	VW						·99	·85	·99	·98	·79	·91	·95
7.	VA							·86	·98	·99	·79	·92	·94
8.	CE								·84	·84	·68	·76	·79
9.	47E									·99	·74	·84	·96
10.	47VA										·75	·89	·94
11.	T-1											·93	·82
12.	T-20												·94
13.	T-100												

TABLE 7: East South Central
Coefficients of Correlation
(272 units: 10 metropolitan areas, 262 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·95	·97	·96	·97	·97	·95	·91	·97	·97	*	·99	·99
2. E		·99	·97	·99	·99	·99	·94	·99	·99	*	·93	·98
3. VEP			·99	·99	·99	·99	·94	·99	·99	*	·94	·98
4. PW				·99	·99	·99	·96	·98	·99	*	·94	·99
5. PMH					·99	·99	·94	·98	·98	*	·96	·97
6. VW						·99	·94	·99	·98	*	·97	·97
7. VA							·96	·99	·99	*	·92	·97
8. CE								·93	·94	*	·91	·93
9. 47E									·99	*	·97	·99
10. 47VA										*	·96	·97
11. T-1											*	*
12. T-20												·97
13. T-100												

* Due to mechanical error, the coefficients involving criterion number 11 (in the computation of this table only) were judged unreliable and are therefore omitted from this table.

TABLE 8: West South Central
Coefficients of Correlation
(289 units: 22 metropolitan areas, 267 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·97	·95	·95	·97	·94	·93	·70	·94	·91	·99	·99	·98
2. E		·99	·98	·99	·99	·98	·80	·97	·96	·97	·95	·98
3. VEP			·99	·99	·99	·99	·81	·99	·97	·96	·96	·96
4. PW				·98	·99	·97	·80	·97	·96	·95	·95	·98
5. PMH					·99	·97	·79	·99	·96	·96	·97	·98
6. VW						·99	·83	·99	·97	·94	·95	·96
7. VA							·86	·98	·99	·94	·91	·95
8. CE								·87	·88	·69	·68	·78
9. 47E									·99	·93	·94	·98
10. 47VA										·91	·91	·95
11. T-1											·96	·98
12. T-20												·99
13. T-100												

TABLE 9: Rocky Mountain
Coefficients of Correlation
(125 units: 9 metropolitan areas, 116 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·99	·98	·99	·95	·98	·93	·88	·89	·96	·98	·91	·98
2. E		·99	·90	·97	·98	·98	·92	·97	·97	·99	·88	·99
3. VEP			·94	·99	·99	·99	·91	·95	·96	·99	·95	·99
4. PW				·93	·97	·97	·92	·98	·97	·98	·99	·95
5. PMH					·99	·98	·93	·98	·99	·98	·94	·99
6. VW						·98	·91	·94	·97	·97	·94	·98
7. VA							·89	·98	·94	·91	·99	·96
8. CE								·87	·89	·88	·85	·90
9. 47E									·96	·92	·99	·97
10. 47VA										·97	·95	·98
11. T-1											·92	·99
12. T-20												·98
13. T-100												

TABLE 10: Pacific Coast
Coefficients of Correlation
(105 units: 12 metropolitan areas, 93 counties)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·99	·99	·99	·99	·99	·99	·98	·98	·97	·99	·99	·99
2. E		·99	·99	·99	·99	·99	·98	·98	·97	·99	·99	·99
3. VEP			·99	·99	·99	·99	·98	·98	·97	·99	·99	·99
4. PW				·99	·99	·99	·98	·98	·97	·99	·99	·99
5. PMH					·99	·99	·98	·98	·97	·99	·99	·99
6. VW						·99	·98	·98	·97	·99	·99	·99
7. VA							·98	·99	·98	·99	·99	·99
8. CE								·98	·98	·98	·98	·99
9. 47E									·99	·98	·98	·99
10. 47VA										·97	·97	·99
11. T-1											·99	·99
12. T-20												·99
13. T-100												

TABLE 11: Standard Metropolitan Areas
Coefficients of Correlation
(166 units)

	2 *E*	3 *VEP*	4 *PW*	5 *PMH*	6 *VW*	7 *VA*	8 *CE*	9 *47E*	10 *47VA*	11 *T-1*	12 *T-20*	13 *T-100*
1. T	·94	·92	·94	·94	·91	·93	·34	·92	·93	·99	·99	·93
2. E		·99	·99	·99	·99	·99	·56	·99	·99	·93	·94	·99
3. VEP			·99	·99	·99	·99	·61	·98	·98	·91	·92	·98
4. PW				·99	·99	·99	·56	·99	·99	·93	·94	·99
5. PMH					·99	·99	·58	·99	·99	·93	·93	·98
6. VW						·99	·63	·98	·98	·90	·91	·97
7. VA							·60	·98	·99	·92	·92	·98
8. CE								·55	·55	·33	·33	·53
9. 47E									·99	·91	·92	·98
10. 47VA										·92	·93	·98
11. T-1											·99	·92
12. T-20												·93
13. T-100												

Standard Metropolitan Areas: Thus far there appears to be a very high degree of co-variation between most of the thirteen criteria, whether their co-variance be viewed from the standpoint of the nation as a whole or from that of regional segments of the whole. The next question was whether high coefficients prevailed among the 166 metropolitan areas as distinct from the 1980 counties which are part of no metropolitan area?

Table 11 portrays the coefficients for the 78 possible pairs of criteria as they vary among the 166 metropolitan areas only. Again, coefficients of ·99 prevail wherever criteria 2, 3, 4, 5, 6, and 7 are involved with each other. Also, every other pair of criteria has a coefficient of at least ·90 except where criterion number 8 is involved. Coefficients involving criterion number 8 vary from a low ·33 to a high of only ·66.

5. SUMMARY

The highest co-variation (whether viewed from the standpoint of the nation as a whole, or of individual census regions, or of the 166 metropolitan areas as a group) appears among criteria numbered 2, 3, 4, 5, 6, and 7 (measuring employment, man-hours, wages, and value added). Whenever these factors are paired among themselves, the resulting coefficients are ·99, with very few exceptions.

The two variables representing 1947 data (criteria numbered 9 and 10), *number of employees, 1947* and *value added, 1947* likewise co-varied so closely that a coefficient of ·99 appeared regardless of whether the two were paired off for the nation as a whole (Table 1), or for the metropolitan areas only (Table 11), or for seven of the nine census regions — only in the *South Atlantic* (Table 6) and the *Rocky Mountain* (Table 9) areas did the coefficient for this pair drop below ·99 and even in these two areas, the drop was slight (·98 in the *South*

Atlantic region and ·96 in the *Rocky Mountain* region).

When any one of the criteria numbered 2 through 7 is correlated with any one of the criteria number 9–13, high coefficients result. There are fewer readings of ·99 than when criteria 2 through 7 are correlated with each other; nevertheless, there are few readings below ·90.

The glaring exception among the the thirteen variables is number 8, *capital expenditures in 1954*. No matter with what other criterion this variable is correlated, whether from a nation-wide view-point, or for that of any one census region, or among the metropolitan areas only, the resulting coefficients are always the lowest in the matrix. This suggests that the locational pattern of manufacturing is changing as some areas are getting "more than their share" of new capital investments.

If high correlations support the assumption upon which this research was started, that variables with high coefficients would tend to produce the same locational pattern when mapped individually, then it would appear that whether one were mapping manufacturing for the nation as a whole, or for any one census region, or for just the standard metropolitan areas, it would be immaterial which of the six criteria numbered 2 through 7 one selected. Criteria numbered 1, 9, 10, 11, 12, and 13 would be almost as good.

6. QUESTIONS

The evidence and conclusions just presented raise several questions:

1. Would more meaningful conclusions have resulted if they had been reached in the framework provided by statistical significance tests?

2. Would there be merit in employing "linkage analysis" (or some type of objective grouping technique) to isolate clusters of similar coefficients?

3. Is there stability through time in these coefficients? Admittedly, the research reported in this article is for just one date — 1954. A next step in investigating this general topic is to see if there is change (and if so how much) in these coefficients over time.

4. How does one determine whether a correlation is "high enough" for the specific problem on which he is working? Conceivably, one scholar might be analyzing a problem in manufacturing geography in which a correlation of ·85 would be "high enough" to warrant substitution of one criterion for another, while another scholar might be working on a problem where a correlation of ·96 would not be "high enough" to warrant such substitution. In other words, how does one determine, for his particular problem, the degree of exactness required?

5. Finally, the authors have some question in their minds about the formula employed for computing coefficients of correlation. The present study used the well known "Pearson product-moment" formula.

However, the data used in this study do not conform to a normal distribution. Rather, the data are preponderantly low values; that is the largest number of entries are areal units with small amounts of manufacturing, a smaller number of entries have moderate amounts of manufacturing, and still fewer entries have large amounts of manufacturing. The question then is: would some other measures of correlation (i.e. rank-order coefficients) yield more

meaningful results than does the Pearson formula? What, for instance, would be revealed by a rank correlation or some normalizing transformation of the variables?

7. CONCLUSION

At this point the authors turn their findings over to colleagues who are more experienced in using statistical techniques. The authors have several reservations in their own minds about the findings based on their data. Nevertheless, for whatever use it may be in spatial analysis of manufacturing, the present article embodies some evidence on correlation of measurements of manufacturing.

REFERENCES

[1] de Geer, Sten. 'Delimitation of the North American Manufacturing Belt' in *Geografiska Annaler*, H. 4, 1927, pp. 247-58.

[2] Strong, H. 'Regions of Manufacturing Intensity in the United States' in *Annals*, Association of American Geographers, Vol. XXVII, 1937, pp. 23-43.

[3] Wright, Alfred, 'Manufacturing Districts of the United States' in *Economic Geography*, Vol. 24, 1938, pp. 195-200.

[4] Roterus, V. 'Value Added by Manufacture and Its Significance' in *Virginia Economic Review*, Vol. I 1938, pp. 1-3.

[5] Alexandersson, G. *The Industrial Structure of American Cities* (Nebraska University Press, 1956, and Stockholm, 1956) pp. 27-91.

[6] McCarty, H. *The Geographic Basis of American Economic Life* (New York, 1940), pp. 482-83.

[7] Thompson, John H. 'A New Method for Measuring Manufacturing' in *Annals*, Association of American Geographers, Vol. XLV, 1955, pp. 416-36.

[8] Zelinsky, W. 'A Method for Measuring Change in the Distribution of Manufacturing Activity: the United States, 1939-1947' in *Economic Geography*, Vol. 34, 1958, pp. 95-126.

[9] Alexander, John W. 'Location of Manufacturing: Methods of Measurement' in *Annals*, Association of American Geographers, Vol. 48, 1958, pp. 20-26.

[10] U. S. Government, Department of Commerce, Bureau of the Census. *Census of Manufactures 1954*

BOOKS RECEIVED

(Listing below does not preclude the review of any book in a later issue.)

Banfield, E.C., *Political Influence*. New York: Free Press of Glencoe, 1961, Pp. v, 354. $6.00.

Crisafulli, V.C., *An Economic Analysis of the Utica-Rome Area*. Utica: Utica College Research Center, 1960, Pp. 294.

Duncan, B., and Hauser, P.M., *Housing a Metropolis—Chicago*. New York: Free Press of Glencoe, 1960, Pp. xxii, 278. $7.50.

Duncan, O., Cuzzort, R.P., and Duncan, B., *Statistical Geography*. New York: Free Press of Glencoe, 1961, Pp. vii, 191.

Edgel, R.L. and Ximenes, V.T., *Income and Employment in New Mexico, 1949–1959*. Albuquerque: Bureau of Business Research: University of New Mexico, 1961, Pp. 76.

Firey, W., *Man, Mind and Land-A Theory of Resource Use*. New York: Free Press of Glencoe, 1960, Pp. 256, $6.00.

Hirschman, A.O., *The Strategy of Economic Development*. New Haven: Yale University Press, 1961, Pp. xiv, 217, $5.00.

Kuznets, S., Miller, A.R., and Easterlin, R.A., *Population Redistribution and Economic Growth-U.S. 1870–1950. II Analyses of Economic Change*. Philadelphia: American Philosophical Society, 1960, Pp. xiv, 289, $5.00.

Myles, J.C., and Doehne, H.A., *Population Projection, Utica-Rome S.M.A 1960–1970*. Utica: Utica College Research Center, 1960, Pp. 30.

New Mexico's Economy in 1960. Albuquerque: Bureau of Business Research: University of New Mexico, 1961, Pp. 61.

Palmer, E.Z., Thomas, D.T., and Polkinghorn, R.S., *City and Regional Wage Comparisons*, University of Nebraska, College of Business Administration, Business Research Bulletin #64, 1960, Pp. 108.

Rodwin, L. (ed.), *The Future Metropolis*, G. Braziller, 1961.

Stewart, W., *West Virginia Redevelopment Plan Webster County*. Charleston: West Virginia Department of Commerce, 1961, Pp. 39.

Triffin, R., *Gold and the Dollar Crisis*. New Haven: Yale University Press, 1961, Pp. xiv, 181, $1.45.

Wagner, P., *The Human Use of the Earth*. New York: Free Press of Glencoe, 1960.

West Virginia Development Agency, *West Virginia Economic Atlas, I and II*. Charleston: West Virginia Department of Commerce, 1960, Pp. 24, 32.

_____, *A Site Selection Security Study*. Charleston: West Wirginia Department of Commerce, 1960, Pp. 20.

Wolff, R.P., *Five Year Forecast of Residential Non-Farm Housing Starts, U.S. 1959–1963*. Coral Gables: Bureau of Business and Economic Research, University of Miami, Florida, 1959, Pp. 25.

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